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Experimental assessment of mixed-mode partition theories for generally laminated composite beams

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ABSTRACT

Three different approaches to partitioning mixed-mode delaminations are assessed for their ability to predict the interfacial fracture toughness of generally laminated composite beams. This is by using published data from some thorough and comprehensive experimental tests carried out by independent researchers (Davidson et al., 2000, 2006). Wang and Harvey's (2012) Euler beam partition theory is found to give very accurate prediction of interfacial fracture toughness for arbitrary layups, thickness ratios and loading conditions. Davidson et al.'s (2000) non-singular-field partition theory has excellent agreement with Wang and Harvey's Euler beam partition theory for unidirectional layups. Although Davidson et al.'s partition theory predicts the interfacial fracture toughness of multidirectional layups reasonably well, overall Wang and Harvey's Euler beam partition theory is found to give better predictions. In general, the singular-field approach based on 2D elasticity and the finite element method gives poor predictions of fracture toughness.

1. Introduction

Delamination is a major concern in the application of laminated composite materials and has attracted the attention of many researchers for decades. Although delamination generally occurs as mixed-mode fracture with all three opening, shearing and tearing actions (i.e. mode I, II and III), 1D delamination has received more attention as it is simpler, still captures the essential mechanics, and also serves as a stepping stone towards the study of general mixed-mode delamination. The expression '1D delamination' means that a delamination propagates in one direction with mode I and mode II action only. Examples of 1D delamination include through-width delamination in double cantilever beams (DCBs), and blisters in laminated composite plates and shells. A central task in studying 1D delamination is to partition the total energy release rate (ERR) G of a mixed-mode fracture into its individual mode I and II ERR components, that is, G_{I} and G_{II}, which govern the propagation of the mixed-mode fracture.

Several relatively well-known partition theories for beam structures are Williams' partition theory [1], Suo and Hutchinson's partition theory [2,3], Davidson et al.'s partition theories [4–6] and Wang and Harvey's partition theories [8–12]. All these theories assume a rigid crack interface, that is, they assume that no relative crack tip separation occurs before crack growth. Therefore these theories effectively consider brittle fracture. It is worth noting that the assumption of a rigid crack interface has profound mechanical implications on mixed-mode partitioning. Some further points regarding this will be given later. Williams' partition theory [1] is based on Euler beam theory, and for rigid interfaces is applicable to midplane delamination in laminated unidirectional (UD) composite beams only. It is often called the 'global partition theory'. Suo and Hutchinson's partition theory [2,3] is based on 2D-elasticity theory and stress intensity factors and is applicable to both midplane delamination and offset delamination (i.e. not on the midplane) in laminated UD composite beams. It is often called the 'local partition theory'. Davidson et al.'s partition theories [4–6] include a singular-field partition theory and a non-singular-field partition theory. Both theories are derived by using a combined analytical and numerical approach based on 2D elasticity with stress intensity factors. Experimental data are also used in the derivation of the non-singular-field partition theory [4–6]. Both are applicable to delamination in laminated composite beams with arbitrary through-thickness location and with arbitrary layup. Wang and Harvey's partition theories [8–12] include an Euler beam partition theory, a Timoshenko beam partition theory, and a partition theory for 2D elasticity. These theories are completely analytical and derived by discovering a fundamentally different and powerful methodology. Stress intensity factors are not used.

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Nomenclature

A_1, A_2, A	extensional stiffness of upper, lower and intact beams
b	beam width
B_1, B_2, B	coupling stiffness of upper, lower and intact beams
D_1, D_2, D	bending stiffness of upper, lower and intact beams
E	Young's modulus
E_{1f}	flexural modulus
G_{I}, G_{II}, G	mode I, mode II and total energy release rate
G_{Ic}, G_{IIc}, G_c	mode I, mode II and total fracture toughness
h_1, h_2, h	thicknesses of upper, lower and intact arms
M_1, M_2	bending moments on upper and lower arms
M_{1B}, M_{2B}	bending moments at crack tip on upper and lower arms
M_c, N_c	concentrated crack tip moment and force
n_1, n_2	numbers of plies in the upper and lower arms
N_1, N_2	axial forces on upper and lower arms
N_{1B}, N_{2B}	axial forces at crack tip on upper and lower arms
β, β'	pure mode II relationships from the first and second set
γ	thickness ratio h_2/h_1
θ, θ'	pure mode I relationships from the first and second set
μ	shear modulus
ν	Poisson's ratio
σ	standard deviation
Ω	mode mix parameter

Abbreviations

CUD	constrained unidirectional
DCB	double cantilever beam
ENF	end-notched flexure
ERR	energy release rate
MMB	mixed-mode bending
MD	multidirectional
SSLB	symmetric single leg bending
UD	unidirectional
UENF	unsymmetric end-notched flexure
USLB	unsymmetric single leg bending

All of them are applicable to delamination in laminated composite beams with arbitrary through-thickness location and with arbitrary layup.

Which of the above partition theories [1–12] can best complete the central task: to partition the total ERR G into G_{I} and G_{II}, and in doing so, predict the fracture toughness? Only measurements from experimental tests are able to answer this question. Although there are numerous experimental investigations reported in literature, the ones in Refs. [5–7,11,13–16] may represent some of the most comprehensive and convincing ones. By using a linear failure locus (found to be a good approximation for the tested composite material), an experimental investigation for delamination in UD laminates is reported in Ref. [15] for the assessment of Williams' partition theory [1] and Suo and Hutchinson's 2D-elasticity partition theory [2,3]. The conclusion of those researchers was that the former agrees with the linear failure locus much better than the latter does. The experimental investigations reported in Refs. [5–7] are for both UD and multidirectional (MD) laminates. No specific failure locus is assumed, and instead a failure locus is experimentally determined in terms of the total critical ERR G_c and G_{II}/G by using the test data for midplane delamination in UD laminates. All the partition theories agree on this particular case and so the failure locus is reliably obtained. Then, the assessment of different partition theories is made against this midplane failure locus for delamination at various through-thickness locations and with various layups. The experimental investigation in Ref. [5] assesses Williams' partition theory [1] and Davidson et al.'s 2D-elasticity singular field and non-singular-field partition theories [4–6]. Quoting from Ref. [5], the conclusions are: (1) "a singular-field-based definition of mode mix will not produce accurate delamination growth predictions for certain composite materials and loadings"; (2) "an alternative definition of mode mix, originally developed by Williams and successfully applied to other composite systems [14–16], is not universally applicable"; (3) the non-singular-field partition theory "would appear to be more appropriate than the classical approach for many current continuous fibre composites." Even more comprehensive experimental assessments are given in Refs. [6,7] for Davidson et al.'s 2D-elasticity singular-field partition theory and non-singular-field partition theory [4–6], including results from various finite element simulations. A large number of UD and MD laminates are tested in different bending and tension configurations. The assessment methodology is the same as that in the study [5], that is, a failure locus is experimentally determined in terms of the total critical ERR G_c and G_{II}/G by testing UD laminates with midplane delamination. Different partition theories are then assessed against this failure locus using test specimens with delamination at various through-thickness locations and with various layups. The assessment concluded that Davidson et al.'s 2D-elasticity non-singular-field partition theory [4–6] provides highly accurate delamination growth predictions for a variety of laminate layups and loadings. Conversely, the 2D-elasticity singular-field partition theory [4–6] is shown to have relatively poor accuracy.

Recently, the authors have made a detailed experimental assessment [11] of Williams' [1], Suo and Hutchinson's [2,3], and Wang and Harvey's [8–11] partition theories using the same methodology and test data as that used in the study in Ref. [15]. It was shown that the predictions from Wang and Harvey's Euler beam partition theory [8–11] have the best agreement with the linear failure locus that was originally suggested in Ref. [15] for the composite material in question, following it extremely closely. The predictions from Wang and Harvey's partition theories for Timoshenko beams and for 2D elasticity, and from Suo and Hutchinson's 2D-elasticity partition theory, are far away from the failure locus, and Williams' partition theory [1] performs much better than them. The very latest work [17] on the topic is also highly regarded. The same assessment methodology to that used in Refs. [5–7] is used (see above). It is shown that Wang and Harvey's Euler beam partition theory [8–11] and Davidson et al.'s non-singular-field partition theory [4–6] have similar performance. Although the authors conclude that none of the current analytical partition theories "are able to predict failure in asymmetric composite laminates", the data presented in the paper shows that both Davidson et al.'s non-singular-field partition theory [4–6] and Wang and Harvey's Euler beam partition theory [8–11] actually show quite reasonable agreement with the midplane failure locus.

In conclusion, from these four independent assessments it appears that both Wang and Harvey's Euler beam partition theory [8–11] and Davidson et al.'s non-singular-field partition theories [4–6] provide the best ERR partitions, G_{I} and G_{II}, which govern the growth of delamination. These two partition theories, however, are derived from very different approaches. The former is based on Euler beam theory and is derived completely analytically, while the latter is based on 2D-elasticity theory and is derived by using a combined analytical, numerical and experimental approach. A detailed explanation is given in Ref. [11] for why Wang and Harvey's Euler beam partition theory [8–11] agrees so well with