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Post-buckling behavior of imperfect laminated composite plates with rotationally-restrained edges

Qingyuan Chen [a], Pizhong Qiao [a,b,*]

[a] State Key Laboratory of Ocean Engineering and School of Naval Architecture, Ocean and Civil Engineering, Shanghai Jiao Tong University, Shanghai 200240, PR China
[b] Department of Civil and Environmental Engineering, Washington State University, Sloan Hall 117, Pullman, WA 99164-2910, USA

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ABSTRACT

The nonlinear governing equations of rotationally-restrained laminated composite plates with imperfection are presented by the Galerkin method, and they are solved by employing the Newton–Raphson method for the post-buckling analysis. The considered laminates are symmetric, and they are loaded in pure in-plane shear or combined in-plane shear and compression. The deformation shape function of the restrained plates is obtained through a linear combination of vibration eigenfunctions of simply supported and clamped beams along either the longitudinal or transverse direction of plates. The validity study shows that the presented method is effective for performing the nonlinear analysis of laminates with all four edges elastically-restrained against rotation. A parametric study is conducted to evaluate the effect of rotational spring stiffness, material properties, and fiber orientation under pure in-plane shear as well as the loading ratio under combined shear and compression on the nonlinear static and post-buckling behavior of rotationally-restrained laminates. The proposed solution for nonlinear static analysis of rotationally-restrained composite plates with imperfection is accurate and effective, as demonstrated by the comparisons with the predictions by the finite element analysis, and combined with the discrete plate analysis technique, it can be potentially applied to post-buckling analysis of FRP structural shapes.

1. Introduction

The simply supported and clamped boundary conditions are two extreme and ideal cases, and the boundary edges are usually elastically restrained by the adjacent structures in reality. Some researchers have studied the elastic large deflection or post-buckling behavior of isotropic and laminated plates with the edges elastically restrained and under compression by the analytical method [1,2], semi-analytical method [3–5], and numerical method [6,7].

Shear post-buckling of composite laminates has attracted less attention than that of laminates under compression, and most of the existing studies were about the laminates with the simply supported or clamped boundary edges, such as the studies by using the analytical method [8], semi-analytical method [9–14], and numerical method [15,16]. Concerning the laminated plates with elastically-restrained edges subjected to shear loading, Quatmann and Reimerdes [17] presented an analytical method for post-buckling behavior of composite fuselage structures under combined compression and shear loading, in which the torsional restraints along the long edges of the plates were considered to simulate the influence of different types of stringers on the post-buckling behavior. Beerhorst et al. [18] also investigated the post-buckling behavior of an infinitely long symmetric and balanced laminate with the longitudinal edges elastically restrained by the torsional springs and under in-plane compression and shear using the analytical method. The above two studies researched the long laminates under shear loading, but the behavior of relatively short laminates under shear loading was not considered. Chia [19] developed a semi-analytical solution for post-buckling analysis of an unsymmetrically-laminated angle-ply rectangular plate under in-plane compression and edge shear. In Chia [19]'s study, the opposite edges of the laminates were assumed to be elastically restrained against rotation to the same degree, and the study only presented the numerical results for post-buckling of the square plates under uniaxial and biaxial compression.

In this paper, a nonlinear static solution for the relative short imperfect symmetric laminates with four edges rotationally-restrained and subjected to the combined shear and compression (as shown in Fig. 1) is presented. The nonlinear governing equations are derived using the Galerkin method, and the

* Corresponding author at: Department of Engineering Mechanics, Shanghai Jiao Tong University, Mulan Building A903, Shanghai 200240, PR China.
E-mail addresses: qiao@sjtu.edu.cn, qiao@wsu.edu (P. Qiao).

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Newton–Raphson method is then used to solve nonlinear problem. This method is based on the semi-analytical method proposed by Zhang and Matthews [9,10], Kosteletos [11] and Romeo and Frulla [12] for the non-linear analysis of symmetrically laminated plates with simply-supported or clamped boundary conditions. In the present analysis, the eigenfunctions of simply-supported and clamped beams are linearly combined (or uniquely weighed) to satisfy the rotationally-restrained boundary conditions. The numerical results for relatively short laminates with various rotational spring stiffness under in-plane shear loading are presented and compared with those from the numerical finite element analysis. Then, a parametric study is conducted to examine the effect of a wide range of parameters on the nonlinear static and post-buckling behavior of rotationally-restrained plates under shear or combined shear and compressive loading.

2. Theoretical formulations

2.1. Governing equation

The laminated composite plate and coordinate system are shown in Fig. 1, and the length and width of the plate are a and b, respectively. The laminated plate is subjected to the in-plane shear N_{xy} and bi-axial compression N_{xx} and N_{yy}; in addition, the plate is elastically restrained along all four edges with the rotational spring stiffness k_1 at $x = 0$ and a, and k_2 at $y = 0$ and b, respectively. The laminate considered is thin (the plate thickness h is much smaller than the in-plane dimensions of the plate), so the classical laminated plate theory is used. The constitutive relations for the laminated plate are expressed as:

$$\begin{Bmatrix} \mathbf{N} \\ \mathbf{M} \end{Bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B} & \mathbf{D} \end{bmatrix} \begin{Bmatrix} \boldsymbol{\varepsilon}^0 \\ \boldsymbol{\kappa} \end{Bmatrix} \tag{1}$$

where $\mathbf{B}$ is called the bending-extension coupling matrix, and it is a zero matrix for the symmetric laminates as considered in this study, and

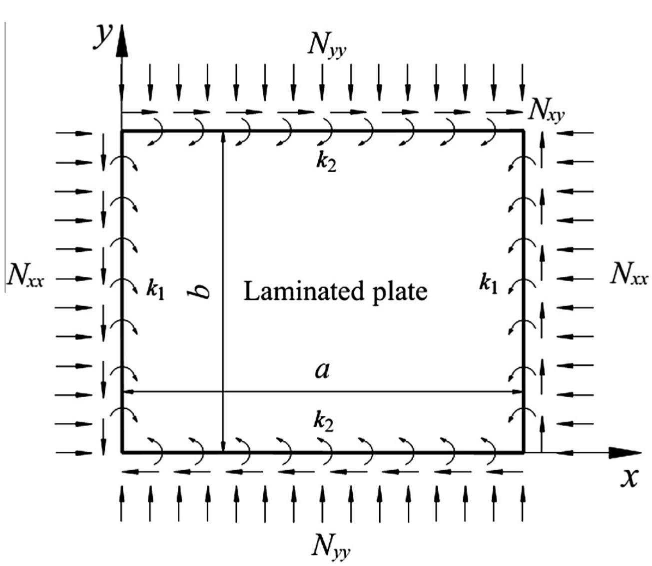

Fig. 1. Geometry of the rotationally-restrained laminates under combined in-plane shear and compression.

$$\begin{aligned} &\mathbf{N} = [N_{xx} \quad N_{yy} \quad N_{xy}]^{\mathrm{T}}, \quad \mathbf{M} = [M_{xx} \quad M_{yy} \quad M_{xy}]^{\mathrm{T}} \\ &\boldsymbol{\varepsilon}^0 = \left[\varepsilon^0_{xx} \quad \varepsilon^0_{yy} \quad \varepsilon^0_{xy}\right]^{\mathrm{T}}, \quad \boldsymbol{\kappa} = \left[-\tfrac{\partial^2 w}{\partial x^2} \quad -\tfrac{\partial^2 w}{\partial y^2} \quad -\tfrac{\partial^2 w}{\partial x \partial y}\right]^{\mathrm{T}} \\ &\mathbf{A} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix}, \quad \mathbf{D} = \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \end{aligned} \tag{2}$$

in which, N_{xx}, N_{yy} and N_{xy} are the in-plane normal and shear forces per unit length; M_{xx}, M_{yy} and M_{xy} are the bending and twisting moments per unit length; ε^0_{xx}, ε^0_{yy} and ε^0_{xy} are the normal and shear strains at the middle surface; w is the transverse deflection of every point (x, y) of the middle surface of the plate; A_{ij} $(i, j = 1, 2, 6)$ are the in-plane extension stiffness, and $D_{ij}(i, j = 1, 2, 6)$ are the bending stiffness (see [20,21]).

Partially inverting Eq. (1) and considering only the symmetric laminates lead to

$$\begin{aligned} &\boldsymbol{\varepsilon}^0 = \mathbf{A}^{-1}\mathbf{N} \\ &\mathbf{M} = \mathbf{D}\,\boldsymbol{\kappa} \end{aligned} \tag{3}$$

The equilibrium equations of a generally layered laminate with imperfection under in-plane loading is given as [12,22]:

$$\begin{aligned} &\frac{\partial N_{xx}}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0 \\ &\frac{\partial N_{xy}}{\partial x} + \frac{\partial N_{yy}}{\partial y} = 0 \\ &\frac{\partial^2 M_{xx}}{\partial x^2} + 2\frac{\partial^2 M_{xy}}{\partial xy} + \frac{\partial^2 M_{yy}}{\partial y^2} + N_{xx}\frac{\partial^2 w}{\partial x^2} + 2N_{xy}\frac{\partial^2 w}{\partial xy} + N_{yy}\frac{\partial^2 w}{\partial y^2} \\ &\quad + N_{xx}\frac{\partial^2 w_0}{\partial x^2} + 2N_{xy}\frac{\partial^2 w_0}{\partial xy} + N_{yy}\frac{\partial^2 w_0}{\partial y^2} = 0 \end{aligned} \tag{4}$$

and the compatibility equation of the laminate with imperfections are reported as [12,22]:

$$\begin{aligned} &\frac{\partial^2 \varepsilon^0_{xx}}{\partial y^2} + \frac{\partial^2 \varepsilon^0_{yy}}{\partial x^2} - \frac{\partial^2 \varepsilon^0_{xy}}{\partial x \partial y} + \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - \frac{\partial^2 w}{\partial x \partial y}\frac{\partial^2 w}{\partial x \partial y} + \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w_0}{\partial y^2} \\ &\quad + \frac{\partial^2 w_0}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - 2\frac{\partial^2 w}{\partial x \partial y}\frac{\partial^2 w_0}{\partial x \partial y} = 0 \end{aligned} \tag{5}$$

Introducing the Airy function $\phi(x, y)$:

$$N_{xx} = \frac{\partial^2 \phi}{\partial y^2}, \quad N_{yy} = \frac{\partial^2 \phi}{\partial x^2}, \quad N_{xy} = -\frac{\partial^2 \phi}{\partial x \partial y} \tag{6}$$

By substituting Eqs. (6) and (3) into Eqs. (4) and (5), respectively, the first two equilibrium equations in Eq. (4) are satisfied spontaneously, and the following equilibrium and compatibility equations in the dimensionless form are obtained [9–12]:

$$\begin{aligned} &\frac{\partial^4 W}{\partial \xi^4} + a_1\frac{\partial^4 W}{\partial \xi^3 \partial \eta} + a_2\frac{\partial^4 W}{\partial \xi^2 \partial \eta^2} + a_3\frac{\partial^4 W}{\partial \xi \partial \eta^3} + a_4\frac{\partial^4 W}{\partial \eta^4} \\ &\quad - a_5\left(\frac{\partial^2 F}{\partial \eta^2}\frac{\partial^2 W}{\partial \xi^2} - 2\frac{\partial^2 F}{\partial \xi \partial \eta}\frac{\partial^2 W}{\partial \xi \partial \eta} + \frac{\partial^2 F}{\partial \xi^2}\frac{\partial^2 W}{\partial \eta^2} + \frac{\partial^2 F}{\partial \eta^2}\frac{\partial^2 W_0}{\partial \xi^2}\right. \\ &\quad \left. - 2\frac{\partial^2 F}{\partial \xi \partial \eta}\frac{\partial^2 W_0}{\partial \xi \partial \eta} + \frac{\partial^2 F}{\partial \xi^2}\frac{\partial^2 W_0}{\partial \eta^2}\right) = 0 \\ &\frac{\partial^4 F}{\partial \xi^4} + b_1\frac{\partial^4 F}{\partial \xi^3 \partial \eta} + b_2\frac{\partial^4 F}{\partial \xi^2 \partial \eta^2} + b_3\frac{\partial^4 F}{\partial \xi \partial \eta^3} + b_4\frac{\partial^4 F}{\partial \eta^4} \\ &\quad + b_5\left(\frac{\partial^2 W}{\partial \xi^2}\frac{\partial^2 W}{\partial \eta^2} - \frac{\partial^2 W}{\partial \xi \partial \eta}\frac{\partial^2 W}{\partial \xi \partial \eta} + \frac{\partial^2 W}{\partial \xi^2}\frac{\partial^2 W_0}{\partial \eta^2}\right. \\ &\quad \left. + \frac{\partial^2 W_0}{\partial \xi^2}\frac{\partial^2 W}{\partial \eta^2} - 2\frac{\partial^2 W}{\partial \xi \partial \eta}\frac{\partial^2 W_0}{\partial \xi \partial \eta}\right) = 0 \end{aligned} \tag{7}$$

in which, the non-dimensional parameters are defined as