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Coupled thermoelectroelastic stress analysis of piezoelectric shells

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ABSTRACT

The paper deals with the sampling surfaces (SaS) method proposed by the authors and its implementation for the three-dimensional (3D) coupled steady-state thermoelectroelastic analysis of laminated piezoelectric shells subjected to thermal loading. The SaS formulation is based on choosing inside the nth layer I_n not equally spaced SaS parallel to the middle surface of the shell in order to introduce the temperatures and displacements of these surfaces as basic shell variables. Such choice of unknowns with the consequent use of Lagrange polynomials of degree $I_n - 1$ in the thickness direction for each layer permits the presentation of the laminated piezoelectric shell formulation in a very compact form. The SaS are located inside each layer at Chebyshev polynomial nodes that allows one to minimize uniformly the error due to the Lagrange interpolation. As a result, the SaS formulation can be applied efficiently to derivation of the analytical solutions for laminated piezoelectric shells, which asymptotically approach the 3D exact solutions of thermoelectroelasticity as the number of SaS goes to infinity.

1. Introduction

Three-dimensional (3D) analysis of laminated piezoelectric plates and shells under thermal loading has received considerable attention during past twenty years (see, e.g. [1,2]). There are at least five approaches to 3D exact solutions of thermoelectroelasticity for piezoelectric plates and shells, namely, the Pagano approach [3,4], the state space approach [5], the power series expansion approach, i.e. the Frobenius method [6], the asymptotic expansion approach, i.e. the perturbation method [7] and the sampling surfaces (SaS) approach [8,9]. The first approach was implemented for piezoelectric plates in contributions [10–14]. The most popular state space approach was utilized efficiently in papers [15–19]. The 3D solution of thermoelectroelasticity for piezoelectric rectangular plates using the asymptotic expansion approach was obtained by Cheng and Batra [20]. The use of the SaS approach for the 3D coupled thermoelectroelastic analysis of laminated piezoelectric plates was carried out by the authors [21]. The exact transient thermal stress analysis of laminated strips made of piezoelectric and magnetostrictive materials was fulfilled by Ootao and his coauthors [22,23].

In the shell formulation, the coefficients of the system of differential equations depend on the thickness coordinate. This implies that the Pagano approach and the state space approach cannot be applied to 3D exact solutions for shells directly. For solving such problem the Frobenius method [24–29] can be employed. It is also possible to artificially divide the shell into a large number of individual layers with the constant coefficients through their thicknesses (see, e.g. [30–32]) following a technique proposed by Soldatos and Hadjigeorgiou [33]. However, the solutions derived via such a technique are not exact, they are approximate. After a close survey of the 3D coupled thermopiezoelectric structure analysis in the open literature, we found that the SaS method had not been applied yet to laminated piezoelectric shells. The present paper is intended to fill the gap of knowledge in this research area.

The SaS formulation was first utilized for the 3D elasticity analysis of laminated composite plates and shells [8,9]. Then, the SaS formulation was extended to the heat conduction theory [34], thermoelasticity [35,36] and electroelasticity [37–39]. According to the SaS concept, we choose arbitrarily located surfaces inside the nth layer parallel to the middle surface of the shell $\Omega^{(n)1}, \Omega^{(n)2}, \ldots, \Omega^{(n)I_n}$ in order to introduce temperatures $T^{(n)1}, T^{(n)2}, \ldots, T^{(n)I_n}$, electric potentials $\varphi^{(n)1}, \varphi^{(n)2}, \ldots, \varphi^{(n)I_n}$ and displacement vectors $\mathbf{u}^{(n)1}, \mathbf{u}^{(n)2}, \ldots, \mathbf{u}^{(n)I_n}$ of these surfaces as basic shell variables, where I_n is the total number of SaS of the nth layer ($I_n \geqslant 3$). Such choice of temperatures, electric potentials and displacements with the consequent use of the Lagrange polynomials of degree $I_n - 1$ in the thickness direction for each layer allows the presentation of governing equations of the SaS thermopiezoelectric shell formulation in a very compact form.

It is worth noting that the developed shell formulation with equally spaced SaS does not work properly with the Lagrange polynomials of high degree because of the Runge's phenomenon [40],

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which can lead to oscillations at the edges of the interval when the user deals with the specific shell metric functions. If the number of equally spaced nodes is increased then the oscillations become even larger. However, the use of Chebyshev polynomial nodes [41] as SaS coordinates allows one to minimize uniformly the error due to the Lagrange interpolation. As a result, the SaS formulation can be applied efficiently to the solution of 3D coupled problems for laminated thermopiezoelectric shells with a specified accuracy utilizing the sufficient number of SaS. This is due to the fact that analytical solutions based on the SaS formulation *asymptotically* approach the 3D exact solutions of thermoelectroelasticity as $I_n \to \infty$.

The origins of the SaS concept can be found in contributions [42,43] in which three, four and five equally spaced SaS are employed. The SaS formulation with the arbitrary number of equispaced SaS is considered in paper [44]. The more general approach with the SaS located at the Chebyshev polynomial nodes was developed later [8,9].

2. Description of temperature and temperature gradient fields

Consider a thick laminated shell of the thickness h. Let the middle surface Ω be described by orthogonal curvilinear coordinates θ_1 and θ_2, which are referred to the lines of principal curvatures of its surface. The coordinate θ_3 is oriented along the unit vector $\mathbf{e}_3(\theta_1,\theta_2)$ normal to the middle surface. Introduce the following notations: $\mathbf{e}_\alpha(\theta_1,\theta_2)$ are the orthonormal base vectors of the middle surface; $A_\alpha(\theta_1,\theta_2)$ are the coefficients of the first fundamental form; $k_\alpha(\theta_1,\theta_2)$ are the principal curvatures of the middle surface; $c_\alpha = 1 + k_\alpha\theta_3$ are the components of the shifter tensor; $c_\alpha^{(n)i_n}(\theta_1,\theta_2)$ are the components of the shifter tensor at SaS defined as

$$c_\alpha^{(n)i_n} = c_\alpha(\theta_3^{(n)i_n}) = 1 + k_\alpha\theta_3^{(n)i_n}, \tag{1}$$

where $\theta_3^{(n)i_n}$ are the transverse coordinates of SaS of the nth layer given by

$$\theta_3^{(n)1} = \theta_3^{[n-1]}, \quad \theta_3^{(n)I_n} = \theta_3^{[n]},$$

$$\theta_3^{(n)m_n} = \frac{1}{2}\left(\theta_3^{[n-1]} + \theta_3^{[n]}\right) - \frac{1}{2}h^{(n)}\cos\left(\pi\frac{2m_n-3}{2(I_n-2)}\right), \tag{2}$$

where $\theta_3^{[n-1]}$ and $\theta_3^{[n]}$ are the transverse coordinates of layer interfaces $\Omega^{[n-1]}$ and $\Omega^{[n]}$ depicted in Fig. 1; $h^{(n)} = \theta_3^{[n]} - \theta_3^{[n-1]}$ is the thickness of the nth layer.

Here and in the following developments, the index n identifies the belonging of any quantity to the nth layer and runs from 1 to N, where N is the number of layers; the index m_n identifies the belonging of any quantity to the inner SaS of the nth layer and runs from 2 to $I_n - 1$, whereas the indices i_n, j_n, k_n describe all SaS of the nth layer and run from 1 to I_n; Latin tensorial indices i, j, k, l range from 1 to 3; Greek indices α, β range from 1 to 2.

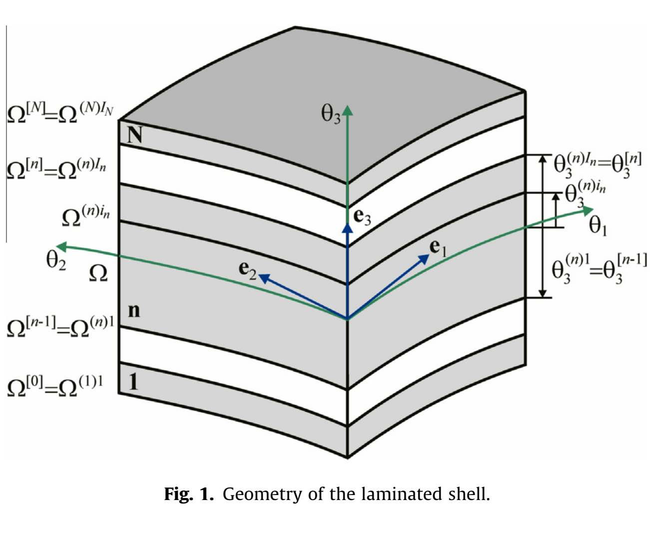

Fig. 1. Geometry of the laminated shell.

Remark 1. It is seen from Eq. (2) that the transverse coordinates of inner SaS $\theta_3^{(n)m_n}$ coincide with coordinates of the Chebyshev polynomial nodes [41]. This fact has a great meaning for a convergence of the SaS method [8,9].

The relation between the temperature T and the temperature gradient $\boldsymbol{\Gamma}$ is given by

$$\boldsymbol{\Gamma} = \nabla T. \tag{3}$$

In a component form, it can be written as

$$\Gamma_\alpha = \frac{1}{A_\alpha c_\alpha}T_{,\alpha}, \quad \Gamma_3 = T_{,3}, \tag{4}$$

where the symbol $(\ldots)_{,i}$ stands for the partial derivatives with respect to coordinates θ_i.

We start now with the first and second assumptions of the proposed thermopiezoelectric laminated shell formulation. Let us assume that the temperature and temperature gradient fields are distributed through the thickness of the nth layer as follows:

$$T^{(n)} = \sum_{i_n} L^{(n)i_n}T^{(n)i_n}, \quad \theta_3^{[n-1]} \leqslant \theta_3 \leqslant \theta_3^{[n]}, \tag{5}$$

$$\Gamma_i^{(n)} = \sum_{i_n} L^{(n)i_n}\Gamma_i^{(n)i_n}, \quad \theta_3^{[n-1]} \leqslant \theta_3 \leqslant \theta_3^{[n]}, \tag{6}$$

where $T^{(n)i_n}(\theta_1,\theta_2)$ are the temperatures of SaS of the nth layer $\Omega^{(n)i_n}$; $\Gamma_i^{(n)i_n}(\theta_1,\theta_2)$ are the components of the temperature gradient at the same SaS; $L^{(n)i_n}(\theta_3)$ are the Lagrange polynomials of degree $I_n - 1$ defined as

$$T^{(n)i_n} = T(\theta_3^{(n)i_n}), \tag{7}$$

$$\Gamma_i^{(n)i_n} = \Gamma_i(\theta_3^{(n)i_n}), \tag{8}$$

$$L^{(n)i_n} = \prod_{j_n \neq i_n} \frac{\theta_3 - \theta_3^{(n)j_n}}{\theta_3^{(n)i_n} - \theta_3^{(n)j_n}}. \tag{9}$$

The use of Eqs. (4), (5), (6) and (8) yields

$$\Gamma_\alpha^{(n)i_n} = \frac{1}{A_\alpha c_\alpha^{(n)i_n}}T_{,\alpha}^{(n)i_n}, \tag{10}$$

$$\Gamma_3^{(n)i_n} = \sum_{j_n} M^{(n)j_n}(\theta_3^{(n)i_n})T^{(n)j_n}, \tag{11}$$

where $M^{(n)j_n} = L_{,3}^{(n)j_n}$ are the derivatives of the Lagrange polynomials, which are calculated at SaS as follows:

$$M^{(n)j_n}(\theta_3^{(n)i_n}) = \frac{1}{\theta_3^{(n)j_n} - \theta_3^{(n)i_n}} \prod_{k_n \neq i_n, j_n} \frac{\theta_3^{(n)i_n} - \theta_3^{(n)k_n}}{\theta_3^{(n)j_n} - \theta_3^{(n)k_n}} \quad \text{for} \quad j_n \neq i_n,$$

$$M^{(n)i_n}(\theta_3^{(n)i_n}) = -\sum_{j_n \neq i_n} M^{(n)j_n}(\theta_3^{(n)i_n}). \tag{12}$$

It is seen from Eq. (11) that the values of the temperature gradient on SaS of the nth layer $\Gamma_3^{(n)i_n}$ are represented as a linear combination of temperatures of SaS of the same layer $T^{(n)j_n}$.

3. Variational formulation of heat conduction problem

The variational equation for the thermal laminated shell is written as

$$\delta J = 0, \tag{13}$$

where J is the basic functional of the heat conduction theory given by