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Linear stability analysis of a three-dimensional viscoelastic liquid jet surrounded by a swirling air stream

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ABSTRACT

A theoretical model is established to investigate the instability of a viscoelastic liquid jet with axisymmetric and non-axisymmetric disturbances, which is moving in a swirling air stream. The dispersion relation is derived by a temporal linear stability analysis. Results show that the three-dimensional viscoelastic liquid jet is more unstable than its Newtonian counterpart when considering the air swirl. The effects of air swirl strength, jet velocity, surface tension, liquid viscosity and gas density on the instability of viscoelastic jet surrounded by a swirling gas are analogous with the example of the Newtonian jet. Note that air swirl is also a stabilizing factor on the instability of the viscoelastic jet. The axisymmetric mode can prevail over the non-axisymmetric when the swirl strength is strong enough, while the non-axisymmetric mode is dominant in a liquid jet with a high liquid Weber number and a low liquid Reynolds number. It is also found that the maximum unstable growth rate of a viscoelastic liquid jet increases as liquid elasticity increases or the time constant ratio decreases. Furthermore, when air swirl velocity is introduced, it is the gas-to-liquid relative axial velocity that governs jet instability in axisymmetric disturbances, while the absolute gas axial velocity is another influencing factor in the non-axisymmetric mode. Finally, the competition between the gas rotating and axial velocities on jet instability is examined. In the case of a relatively smaller resultant gas velocity, the effect of the gas axial velocity can prevail over that of the air swirl when the gas velocity ratio is larger in the non-axisymmetric mode, whereas it can always exceed the effect of the air swirl in the axisymmetric mode. For a larger resultant gas velocity, the effect of the gas axial velocity is predominant only at a large gas velocity ratio in the axisymmetric mode.

1. Introduction

The liquid jet is widely used in various practical applications [1–3], such as in diesel engines, gas turbines, liquid fuel rocket engines and spray drying. Considerable research is focused on better methods for atomizing unstable liquid jets.

The study of unstable liquid jets dates back to Rayleigh's pioneering work [4,5]. He conducted a temporal linear stability analysis for an inviscid cylindrical liquid jet moving in vacuum. The results showed that only axisymmetric disturbances can grow and dominate the breakup of a jet, which unfortunately did not agree with the experiments [6–8], which indicated that in special situations non-axisymmetric disturbances can prevail over axisymmetric disturbances. Weber [9] implemented the study of a viscous liquid jet surrounded by an inviscid gas medium, which gave a detailed description of the instability of a Newtonian liquid jet. But Weber's results were limited to axisymmetric disturbances. There have been many studies investigating the mechanisms of jet instability since Weber's work [10–18]. Although the effects of flow conditions, fluid properties and nozzle geometry on jet instability in the axisymmetric mode can be understood through these studies, these theories do not explain the observation of growing non-axisymmetric disturbances.

In previous studies examining jet instability [12,19–21], it is accepted that the axisymmetric mode is always more dangerous than non-axisymmetric. However, a more general investigation of the non-axisymmetric mode may offer a satisfactory explanation for the results of the experiments previously mentioned [6–8]. To determine the physical mechanism of a non-axisymmetric mode, some researchers established different theoretical models to predict its growing process [22–29]. Liu and Liu [22,23] carried out a linear stability analysis for viscoelastic liquid jets when the ambient gas medium was assumed to be inviscid. For the parametric values considered in their paper, they found that the unstable growth rate of non-axisymmetric disturbances can exceed that of axisymmetric disturbances at high liquid Weber number. But their results were limited to the case of small wave number, and the effect of liquid viscosity on the transition between these two disturbances was neglected. Although Lin and Webb [24], Avital [25],

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Ibrahim [26], Yang [27] failed to discover the phenomenon observed by the experiments [6–8], Li [28] successfully proved that the non-axisymmetric mode becomes a dominant mode when the liquid Weber number is high. Ruo et al. [29] gave the boundaries between these two modes (the axisymmetric and non-axisymmetric modes), and executed the experimental conditions which can cause the predominance of non-axisymmetric disturbances clearly.

It should be noted that all the models established in the above studies describe jet instability in the absence of air swirl. Liao et al. [30] examined the effect of air swirl on the instability of a three-dimensional Newtonian liquid jet. Their results showed that the Newtonian jet becomes more stable by increasing the air swirl, whether the trend is the same for the viscoelastic jet is still unknown. Thus, there is a need to investigate the instability of viscoelastic liquid jet moving in a swirling air stream. In addition, it is interesting to explore the elastic – as well as other fluid parametric – effects on jet instability when air swirl is introduced. Note that the transition between the axisymmetric and non-axisymmetric modes is also discussed in this paper. Finally, the competition between the gas axial and rotating velocities on jet instability is researched.

It is worth mentioning that the linear stability analysis can predict the initial growth of disturbances with small amplitudes, but it is unable to explore jet deformation due to the underlying assumption of infinitesimal perturbation magnitude. The viscoelastic jet generates breakup retardation and forms a structure of drops connected by thin ligaments with increasing deformation. Eventually, the jet breakup slows down and generates very large breakup lengths because of the strong nonlinear behavior existing in viscoelastic fluids [31]. In this sense, a nonlinear analysis may help to fully explain the breakup behavior of viscoelastic liquid jet, which will be studied in the future. Note that although the present work is confined to the linear scope, it can also give some insight into the unstable behavior of a viscoelastic jet at the initial stage of disturbance growth. Thus, it is hoped that the present study will contribute to further investigation of the instability of viscoelastic liquid jets.

A three-dimensional viscoelastic liquid jet, which is subjected to a swirling gas medium, is considered in the present study. The dispersion relation is obtained using a linear stability analysis. Finally, the effects of flow conditions on the instability of liquid jet with and without air swirl are investigated by solving the dispersion relationship in the temporal mode.

2. The relationship of derived dispersion

Consider an incompressible infinitely long viscoelastic liquid jet moving in a coaxial swirling air stream. In terms of cylindrical coordinates (r,θ,x), the r-axis is normal to the centerline of liquid jet, θ-axis is in the azimuthal direction, and x-axis is along the moving direction of the liquid jet flow. By neglecting the gravity, the governing equations of liquid jet, which are the conservation laws of mass and momentum, can be expressed as follows:

$$\nabla \cdot \boldsymbol{v} = 0, \tag{1}$$

$$\rho\left(\frac{\partial}{\partial t} + \boldsymbol{v}\cdot\nabla\right) v = -\nabla p + \nabla\cdot\boldsymbol{\tau}, \tag{2}$$

where ρ is the liquid density, t is the time, $\boldsymbol{v}$ is the liquid velocity vector, which can be expressed as (v_r, v_θ, v_x), p is the liquid pressure, and $\boldsymbol{\tau}$ is the stress tensor of liquid.

The rheological equation of state relating the stress tensor to the velocity field is the Oldroyd B-constitutive equation [32–34], which is written in the objective reference frames as given below. It describes the fluid viscoelastic behavior:

$$\boldsymbol{\tau} + \lambda_1 \frac{\mathrm{D}\boldsymbol{\tau}}{\mathrm{D}t} = \eta_0\left(\dot{\boldsymbol{\gamma}} + \lambda_2 \frac{\mathrm{D}\dot{\boldsymbol{\gamma}}}{\mathrm{D}t}\right), \tag{3}$$

where

$$\dot{\boldsymbol{\gamma}} = \nabla\boldsymbol{v} + (\nabla\boldsymbol{v})^T, \tag{4}$$

$$\dot{\boldsymbol{\omega}} = \nabla\boldsymbol{v} - (\nabla\boldsymbol{v})^T, \tag{5}$$

$$\frac{\mathrm{D}\boldsymbol{\tau}}{\mathrm{D}t} = \frac{\partial\boldsymbol{\tau}}{\partial t} + (\boldsymbol{v}\cdot\nabla)\boldsymbol{\tau} + \frac{1}{2}(\dot{\boldsymbol{\omega}}\cdot\boldsymbol{\tau} - \boldsymbol{\tau}\cdot\dot{\boldsymbol{\omega}}), \tag{6}$$

$$\frac{\mathrm{D}\dot{\boldsymbol{\gamma}}}{\mathrm{D}t} = \frac{\partial\dot{\boldsymbol{\gamma}}}{\partial t} + (\boldsymbol{v}\cdot\nabla)\dot{\boldsymbol{\gamma}} + \frac{1}{2}(\dot{\boldsymbol{\omega}}\cdot\dot{\boldsymbol{\gamma}} - \dot{\boldsymbol{\gamma}}\cdot\dot{\boldsymbol{\omega}}), \tag{7}$$

where $\dot{\boldsymbol{\gamma}}$ is the rate of strain tensor, $\dot{\boldsymbol{\omega}}$ is the vorticity tensor, D/Dt is the co-rotational derivative, η_0 is the zero shear viscosity, λ_1 is the stress relaxation time, λ_2 is the deformation retardation time.

Similarly, the governing equations of incompressible inviscid gas are:

$$\nabla\cdot\hat{\boldsymbol{v}} = 0, \tag{8}$$

$$\rho_g\left(\frac{\partial}{\partial t} + \hat{\boldsymbol{v}}\cdot\nabla\right)\hat{\boldsymbol{v}} = -\nabla p_g, \tag{9}$$

where ρ_g is the gas density, $\hat{\boldsymbol{v}}$ is the gas velocity vector, which can be expressed as $(\hat{v}_r, \hat{v}_\theta, \hat{v}_x)$, p_g is the gas pressure.

Because the jet surface is subjected to small disturbances after the liquid jet issues from the nozzle, the equation describing the surface of the liquid jet can be expressed as follows:

$$R = a + \varepsilon\exp(-i\omega t + in\theta + ikx), \tag{10}$$

where a is the undisturbed jet radius, ε is the initial amplitudes of disturbances, ω is the amplification factor for a disturbance of wavelength λ, n represents the disturbance mode, and k is the axial wave number of disturbances which can be expressed as $2\pi/\lambda$. It is worth mentioning that the disturbances are two-dimensional, and the deformation on the jet surface is axisymmetric when $n = 0$. The so-called axisymmetric mode ($n = 0$), is shown in Fig. 1a. When $n = 1$, as is shown in Fig. 1b, the disturbances are three-dimensional, and the jet surface becomes asymmetric in this situation. This mode is called the 'non-axisymmetric mode'. The cross section and inter-

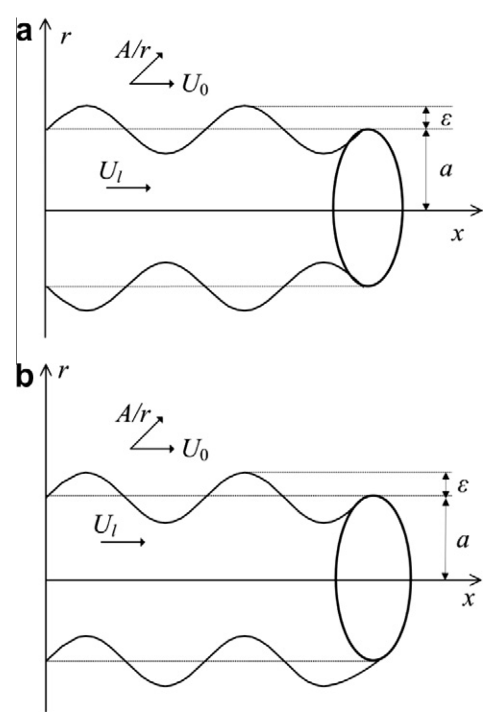

Fig. 1. Schematic of a liquid jet with (a) axisymmetric and (b) non-axisymmetric disturbances.