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# Calculation of interlaminar shear stresses in laminated shallow shell panel using refined higher order shear deformation theory

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ABSTRACT

In general, calculation of inter-laminar shear stresses in laminated shells using 2D finite element models involves cumbersome post processing techniques. This paper presents a simple and efficient method for accurate evaluation of transverse shear stresses in laminated composite shallow shells by using a displacement based $C^0$ 2D FE model derived from refined higher order shear deformation theory (RHSDT) and a least square error (LSE) method. The theory satisfies the inter-laminar shear stress continuity conditions at the layer interfaces and zero transverse shear stress conditions at the top and bottom of the plate. The effect of three curvature terms in the strain components of composite shells is also considered by following the Sander's approximations. In order to overcome the problem of $C^1$ continuity of transverse displacement encountered at the time of FE implementation of the present shell theory (RHSDT), the first derivatives of transverse displacement have been replaced by a independent $C^0$ variables. The LSE method is applied at the post-processing stage, after in-plane stresses are calculated by using the present FE model based on RHSDT. Thus the proposed method is quite simple and efficient compared to the usual method of integrating the 3D equilibrium equations for calculation of transverse stresses in laminated composite shells. Accuracy of the method is demonstrated in the numerical examples through comparison of the present results with those obtained from different models based on refined higher order shear deformation theory (RHSDT), higher order shear deformation theory (HSDT), exact analytical and 3D elasticity solutions.

## 1. Introduction

Laminated composite shell structures are mostly used in several engineering applications; especially in civil, mechanical, aerospace, automobile, marine, nuclear and others. These composite materials are becoming more popular because of their excellent properties such as high strength to weight and high strength to stiffness ratio. However, composite materials are weak in shear due to their low shear modulus compared to extensional modulus. Thus the effect of shear deformation is quite significant which may lead to failure. Moreover, the accurate evaluation of transverse shear stresses by using 2D FE models available in the literature is quite cumbersome. Therefore, there is a genuine requirement for the development of an appropriate shell model for the accurate prediction of transverse shear stresses within the framework of 2D analysis.

In order to properly design laminated composite shell structures, various shell theories and related numerical procedures have been developed. Koiter [1] has stated the following Koiter's recommendation (KR) on 2D modeling of traditional isotropic shells on the bases of energy consideration: "a refinement of Love's first approximation theory is indeed meaningless, in general, unless the effects of transverse shear and normal strains (stresses) are taken into account at the same time." Subsequently Carrera [2,3] investigated the transverse normal strain effects on the bending and vibration of multilayered plates and shells and proposed an amendment of Koiter's recommendation (KR1): "any refinements of classical models are meaningless, in general, unless the effects of inter-laminar continuous transverse shear and normal stresses are both taken into account in a multilayered plate/shell theory."

However, most of the available refinements of classical theories [such as CLT: classical lamination theory, which is based on Kirchhoff's assumptions, and FSDT: first order shear deformation theory, which is based on the so called Reissner–Mindlin assumptions] that have been proposed for homogeneous (one-layered) and multilayered (anisotropic) plates and shells do not account for transverse normal strain ($\varepsilon_z$). The main causes of violating Koiter's recommendation (KR) lie in the intrinsic coupling experienced by isotropic and orthotropic materials between in-plane stresses

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$(\sigma_x, \sigma_y)$ and transverse strain $(\varepsilon_z)$ and vice versa. Due to these couplings, it is more difficult to obtain and solve the differential equations that govern the static and dynamic behaviors of plate/shell structures compared to those plate/shell theories that ignore $\varepsilon_z$. In addition to that, it is difficult to develop multilayered plate/shell theories that are able to a priori fulfill inter-laminar equilibria for both transverse shear $(\sigma_{xz}, \sigma_{yz})$ and normal stresses $(\sigma_z)$ [4].

Most of the classical theories developed for thin elastic shells are based on the Love–Kirchhoff's assumptions. In these theories, the normal to the mid-plane before deformation remains normal and straight after deformation i.e. the effect of transverse shear deformation is neglected. The application of these theories in analyzing laminated composite shells where shear deformation is very significant, may lead to errors in the calculation of deflections, stresses and natural frequencies.

In order to include the effect of transverse shear deformation, Reissner [5] and Mindlin [6] developed the First order shear deformation theory (FSDT) considering the uniform transverse shear strain across the thickness of the plate. This theory required shear correction factors to account for the parabolic distribution of the transverse shear stresses. Love has proposed first approximation theory [7] to study the static and free vibration response of thin elastic shells but this theory is found to be inconsistent with respect to rigid body motions. Later, Sanders has removed this inconsistency by proposing an improved first approximation theory for thin shells [8]. In Sanders' theory all strains vanish for small rigid-body motions of the shell hence Sanders' theory is more close to actual behavior of shells than Love's theory.

Hildebrand et al. [9] and Reissner [10] have considered the effects of transverse shear and normal stresses in developing the theory for thin elastic shells. Ambartsumyan [11] was the first who developed the thin shell theory for the analysis of laminated orthotropic material. In this theory, the Reissner–Mindlin theory for thin elastic shell is extended to the layered anisotropic shells. The bending–stretching coupling effect due to unsymmetric lamination scheme in composite laminates is also included in this theory.

Further improvement in the modeling of the transverse shear deformation across the thickness, higher order shear deformation theories (HSDT) are proposed by many researchers [12–31]. Ambartsumyan [12] was the first who proposed a shear deformation theory which ensures the transverse shear stress continuity conditions across the laminate interfaces and accounts for the parabolic distribution of the transverse shear stresses across the thickness of the composite laminate. Hsu and Wang [13] and Dong and Tso [14] have considered the effect of transverse shear deformation and transverse isotropy to study the response of cylindrical shells. Yang [15] has developed a higher-order shell element with three constant radii of curvature namely, two principal radii, orthogonal to each other and one twist radius. The displacement field $u$, $v$ and $w$ are expressed as the products of one-dimensional Hermite interpolation formulae. Whitney and Sun [16,17] have presented the higher-order shell theory considering the transverse shear deformation and the transverse normal strain, as well as the expansion. However, expressing the transverse displacement field as a function of thickness coordinate will introduce an additional dependent unknown into the theory which, results in a cumbersome and computationally expensive higher-order shell theory. Bhimaraddi [18] also proposed a higher order displacement model which accounts for in-plane inertia, rotary inertia and shear deformation effects on the dynamic response of isotropic cylindrical shells.

Reddy and Liu [19] presented a simple higher-order shear deformation theory (HSDT) by modifying Sander's theory, for the analysis of laminated shells. It contains the same number of dependent unknowns as in the first-order shear deformation theory (FSDT). In this theory, the displacements of the middle surface are expanded as cubic functions of the thickness coordinate and the transverse displacement is assumed to be constant through the thickness. This displacement field leads to the parabolic distribution of the transverse shear stresses (and zero transverse normal strain) and no shear correction factors are used. Subsequently, Mallikarjuna and Kant [20] have also developed a refined shell theory for the analysis of fibre-reinforced composite and sandwich shells. This theory is based on a higher-order displacement model and three-dimensional Hooke's law which gives a realistic cross-sectional deformation. Fraternali and Reddy [21] proposed a penalty finite element model based on layer-wise theory [22] for analyzing the laminated composite shells which can be used to calculate both the displacements and stresses accurately. This theory also predicts the accurate through-thickness distribution of inter-laminar shear stresses which otherwise would have been very difficult to obtain by using a usual two-dimensional displacement based finite element model.

Huang [23] presented a third order shell theory based on Reddy's parabolic shear strain field. In this theory, he has again introduced the shear correction factor to incorporate the continuity of inter-laminar transverse shear stresses. Shu and Sun [24] have also presented an improved higher-order theory for laminated composite plates which satisfy the shear stress continuity condition across each layer interface and includes the effects of material and ply-up patterns on the displacement field. Liew and Lim [25] modified the simple higher order theory proposed by Reddy and Liu [19] by incorporating the Lame parameter $(1 + z/R_x)$ and $(1 + z/R_y)$ to account for the transverse strains. Xiao-ping [26] presented a simple higher-order shear deformation theory for laminated composite shallow shells using Love's first-order geometric approximation and Donnell's simplification. This theory contains the same number of dependent unknowns as in the first order shear deformation theory. This theory also accounts the parabolic distribution of the transverse shear stresses through the thickness of the shell and continuity of transverse shear stresses across each layer interfaces. This theory predicts more accurate responses than first-order theory and some higher-order theories without using the shear correction factor. However, these theories [19,26] demand the $C^1$ continuity of transverse displacements during their finite element implementations. Cho et al. [27] also proposed an efficient higher order shell theory for symmetric laminated composites. In this theory, the in-plane displacement fields are obtained by superposing a global cubically varying displacement field on a zig-zag varying displacement field. This theory satisfies the transverse shear stress continuity condition at layer interfaces and zero transverse shear stress condition at the free surface. In addition to that, they also calculated the accurate through-thickness distribution of transverse stresses for cylindrical shell in the post processing stage after calculating the in-plane displacements based on the first order shear deformation shell theory [28].

Kant and Khare [29] proposed a flat facet quadrilateral composite shell element based on higher-order theory for the analysis of thin-to-thick laminated plates and shells. It is also been observed that among all the higher order theories discussed above, only the theory proposed by Yang [15] account for twist curvature $(1/R_{xy})$, which is essential while analyzing the hyper and conoidal shells. Pradyumna and Bandyopadhyay [30] also studied the behavior of laminated composite shells by extending the higher-order shear deformation theory developed by Kant and Khare [29] considering three radii of curvature. However, this theory contains some nodal unknowns which do not have any physical significance, so incorporation of appropriate boundary conditions becomes a problem.

Bhimaraddi [31] presented three-dimensional elasticity solution for static response of laminated doubly curved shallow shells on rectangular plan-form by assuming the ratio of shell thickness to its middle surface radius as negligible as compared to unity.