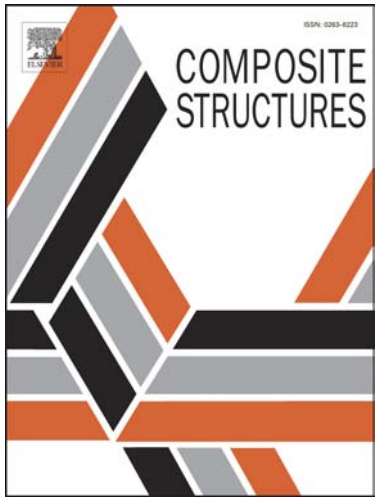

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Some closed-form solutions of functionally graded beams undergoing nonuniform torsion

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Abstract

Torsion of linearly elastic isotropic beams, with both cross-sectional and axial inhomogeneities, is analyzed. Twist (rate of torsional rotation along the beam axis) and warping of cross-sections are not uniform if arbitrary axial variations of elastic properties are considered. Composite beams undergoing nonuniform torsion are commonly investigated by finite and boundary element methods. New closed-form solutions are found in the present paper, by detecting axial distributions of longitudinal and shear moduli inducing an axially uniform warping field. The warping is evaluated by Saint-Venant beam theory, while twist and axial distribution of shear moduli are inversely proportional. Coordinate-free expressions of displacement, normal and shear stress fields are given for simply and multiply connected cross-sections. Exact solutions are obtained for elliptic and equilateral triangle beams, by assuming exponentially graded longitudinal and shear moduli. New benchmarks for numerical analyses are thus also provided.

Key words: Isotropic elasticity, nonuniform torsion, functionally graded materials, analytical modeling, multiply connected cross-sections.

1 Introduction

Torsion of beams is a classical topic of elasticity theory [1–7]. Exact solutions are known for special cross-section shapes and for beams subjected to uniform twisting moment along the axis, see e.g. [8–10]. In this context, twist and