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Effects of constructing different unit cells on predicting composite viscoelastic properties

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ABSTRACT

A computational algorithm of volume average for three-dimensional unit cell model is established to predict the viscoelastic properties of composite materials consisting of linear viscoelastic matrix and transversely isotropic elastic fibers. Several repeating unit cells (RUCs) are constructed with square, hexagonal and random fiber packing to compare the predicted results of composite viscoelastic properties. Proper periodic boundary conditions and necessary physical constraints which are used to stop rigid body motions of the RUCs are implemented. The influences of the ways to construct the RUCs on the predicted results of composite viscoelastic properties are discussed. Some interesting remarks are drawn from the present study.

1. Introduction

Due to the superior properties over conventional engineering materials, composite materials are widely used in conventional and novel areas, including aerospace, civil, electronic and medical engineering. However, many polymer composites always exhibit viscoelastic behavior. The magnitude of the stress components is a function of the deformation history which depends on strain, strain rate, temperature and time [1]. It is difficult to be measured or evaluated conveniently. Proper methodologies should be forwarded to determine this characteristic for engineering as viscoelastic property is a desired property in many circumstances [2]. Consequently, it has been an active research area for many years to research the viscoelastic properties of the composites.

Micromechanical methods provide efficient tools to evaluate the behavior of the composite materials. Many researchers have devoted considerable effort to characterize macro-mechanical properties of composites by using micromechanics modeling. In order to obtain closed-form solutions for composite effective elastic properties in terms of the constituent properties and their volume fractions, many analytical models have been developed, including the concentric cylinder model (CCM) [3–5], the generalized self-consistent method (GSCM) [6–9] and the Mori–Tanaka (M–T) method [10–12]. Based on the homogenization technique and the average of the constituent properties, these methods have been extensively used in the linear analysis for composite structures. However, they neglect the local stresses and strain concentrations within the constituent materials and usually yield an overestimation of the composite nonlinear behavior. In order to overcome these shortcomings and obtain a better prediction of the composite nonlinear response, a few of semi-analytical methods have been developed. The microstructure of a composite material is represented by a repeating unit cell, which can be subsequently partitioned into a number of subregions [13]. The method of cells (MOC) [14] and the generalized method of cells (GMC) [15,16] are powerful semi-analytical methods to approximate the composite effective behavior. In the GMC, the subcell displacement vector is assumed to be linearly expanded in terms of the local subcell coordinates. In the high-fidelity-generalized method of cells (HFGMC) proposed by Aboudi et al. [17], the subcell displacement fields are expanded using second-order approximations. It has been shown that the composite nonlinear response and the gradients in the local fields are accurately predicted [18,19], as compared with results of finite element analysis (FEA). These semi-analytical methods have a distinct advantage over the analytical methods in the solution of local fields where the spatial variations are better resolved; however, the computational time increases rapidly if more details of the nonlinear effects in the local fields are taken into account. The exact solutions of the local fields in the constituent materials can be obtained by fully numerical methods such as FEA, where the micromechanics models are established with detailed fiber geometry and arrangements.

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Several works have been reported on the boundary conditions imposed on the RUC in the FEA [20–23]. The purpose of this paper is to research the effects of the ways to construct the RUCs on the predicted results of composite properties.

In this work, viscoelastic properties of unidirectional fiber-reinforced composites consisting of linear viscoelastic matrix and transverse isotropic elastic fibers are investigated by using three-dimensional unit cell finite element technique. Several RUCs are established with square, hexagonal and random fiber packing. Proper periodic boundary conditions and necessary physical constraints to stop rigid body motions of the RUCs are implemented. The influences of the ways to construct the RUCs on the predicted results of composite viscoelastic properties are discussed. Some interesting remarks are drawn from the present study. Although the analysis is carried out for the linear viscoelastic materials, the outcomes of this work can be suitable for other viscoelastic composites.

2. Mechanical models of composite constituents

Most polymers as the matrix of composite materials exhibit viscoelastic behaviors. That is a combination of viscous and elastic responses to external forces. In this work, the polymer matrix is assumed to follow an isotropic viscoelastic behavior determined by a given set of material parameters. These material constants are assumed to be independent of temperature. The relaxation modulus of the matrix is given by a Prony series expansion:

$$E(t) = E^{\infty} + (E^{u} - E^{\infty})\sum_{i=1}^{N} W_i \exp(-t/\tau_i), \tag{1}$$

where, τ_i is the stress relaxation time, E^{∞} is the fully relaxed modulus, E^{u} is the unrelaxed modulus, W_i is weight factor, and t is the reduced time.

As Poisson's ratio υ^m is a constant for the relaxation process, the shear and bulk moduli of the linear viscoelastic matrix material can be defined by

$$G(t) = \frac{E(t)}{2(1+\upsilon^m)} \tag{2}$$

$$K(t) = \frac{E(t)}{3(1-2\upsilon^m)}. \tag{3}$$

The fibers usually behave as elastic materials in the working temperature of polymer–matrix composites [24]. The carbon fiber is considered as the transversely isotropic elastic material, and is thus modeled by the generalized Hooke's law. Parameters of resin and fiber used in the numerical example are given in Table 1.The stress relaxation times and weight factors of the epoxy resin are shown in Table 2.

In the case of composite materials, stress can be expressed as a function of strain and time given by:

$$\sigma = f(\varepsilon, t). \tag{4}$$

Table 1
Material properties of fiber and resin for the numerical example [30].

Property	Carbon fiber	Property	Epoxy resin
E_1^f (GPa)	207	E^u (GPa)	3.2
E_2^f (GPa)	20.7	E^{∞} (GPa)	0.031
υ_{12}^f	0.2	υ^m	0.35
υ_{23}^f	0.3		
G_{12}^f (GPa)	27.6		
G_{23}^f (GPa)	7.96		
V_f	0.6		

Table 2
Relaxation times and weight factors of epoxy resin [30] (T = 30 °C).

i	W_i	τ_i (min)
1	0.059	29.2
2	0.066	2.92e3
3	0.083	1.82e5
4	0.112	1.1e7
5	0.154	2.83e8
6	0.262	7.94e9
7	0.184	1.95e11
8	0.049	3.32e12
9	0.025	4.92e14

Under small deformation assumption the linear viscoelastic constitutive equation for predicting the relaxation of the stress in the composite structures can be expressed by the following hereditary integral:

$$\langle \sigma_{ij}(t)\rangle = \int_0^t C_{ijkl}(t-\tau)\frac{d\langle \varepsilon_{kl}(\tau)\rangle}{d\tau}d\tau, \quad i,j,k,l = 1,2,3. \tag{5}$$

This can be rewritten in the Voigt vector form as:

$$\begin{Bmatrix} \langle\sigma_{11}(t)\rangle \\ \langle\sigma_{22}(t)\rangle \\ \langle\sigma_{33}(t)\rangle \\ \langle\sigma_{12}(t)\rangle \\ \langle\sigma_{13}(t)\rangle \\ \langle\sigma_{23}(t)\rangle \end{Bmatrix} = \int_0^t \begin{bmatrix} C_{1111} & C_{1122} & C_{1133} & C_{1112} & C_{1113} & C_{1123} \\ C_{2211} & C_{2222} & C_{2233} & C_{2212} & C_{2213} & C_{2223} \\ C_{3311} & C_{3322} & C_{3333} & C_{3312} & C_{3313} & C_{3323} \\ C_{1211} & C_{1222} & C_{1233} & C_{1212} & C_{1213} & C_{1223} \\ C_{1311} & C_{1322} & C_{1333} & C_{1312} & C_{1313} & C_{1323} \\ C_{2311} & C_{2322} & C_{2333} & C_{2312} & C_{2313} & C_{2323} \end{bmatrix} \times \begin{Bmatrix} d\langle\varepsilon_{11}(\tau)\rangle/d\tau \\ d\langle\varepsilon_{22}(\tau)\rangle/d\tau \\ d\langle\varepsilon_{33}(\tau)\rangle/d\tau \\ d\langle\varepsilon_{12}(\tau)\rangle/d\tau \\ d\langle\varepsilon_{13}(\tau)\rangle/d\tau \\ d\langle\varepsilon_{23}(\tau)\rangle/d\tau \end{Bmatrix} d\tau, \tag{6}$$

where, $\langle\cdot\rangle$ represents the averaged value of the component. $C_{ijkl}(t)$ is a 6×6 matrix of stiffness tensor coefficients for a heterogeneous material in general. For a transversely isotropic viscoelastic material as an example, the number of unknown independent coefficients is five. To determine these coefficients, an inverse analysis of micromechanics unit cell is performed. Stress analysis of the unit cell is carried out for six independent load cases using FEA. Each load case can get a set of six independent equations and the stiffness coefficients $C_{ijkl}(t)$ can be solved with such load cases. For every load case, the data obtained from the analysis is conducted further using a volume averaging scheme at each time step to get time-dependent averaged stresses and strains, i.e.:

$$\begin{cases} \langle\sigma_{ij}(t)\rangle = \frac{1}{\Omega}\int_{\Omega}\sigma_{ij}(t)d\Omega = \frac{1}{\Omega}\sum_{n=1}^{m}\sigma_{ij}^{n}(t)\Theta_n \\ \langle\varepsilon_{ij}(t)\rangle = \frac{1}{\Omega}\int_{\Omega}\varepsilon_{ij}(t)d\Omega = \frac{1}{\Omega}\sum_{n=1}^{m}\varepsilon_{ij}^{n}(t)\Theta_n \end{cases}, \tag{7}$$

where, Ω is the volume of the cell, m is the total number of elements, Θ_n is the volume of the nth element, $\sigma_{ij}^{m}(t)$ and $\varepsilon_{ij}^{n}(t)$ are the stress and strain of that element, respectively.

3. Micromechanical models

In a real unidirectional fiber reinforced composite, the fibers are arranged randomly. Usually, the actual cross-section of the