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Review

A new scaled boundary finite element formulation for the computation of singularity orders at cracks and notches in arbitrarily laminated composites

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ABSTRACT

A new formulation of the scaled boundary finite element method (SBFEM) is presented for the static analysis of composites in the framework of classical laminated plate theory. In the SBFEM, the domain is described by the mapping of its boundary with respect to a scaling centre. Therefore, only the boundary needs to be discretised. A local coordinate system is introduced, where a scaling coordinate measures the distance from the scaling centre to the boundary and the other coordinate describes the circumferential direction along the boundary. The displacements are approximated as products of displacement shape functions and unknown functions of the scaling coordinate. Via the virtual work principle, a system of ordinary differential equations for the determination of the unknown displacement functions is obtained, which can be solved in a closed-form analytical manner. Element stiffness matrices for bounded and unbounded domains can be computed, using appropriate subsets of the solution. In the case of cracked composites, the SBFEM enables the effective and precise calculation of singularity orders of stresses, if the scaling centre is selected at the crack tip. Numerical examples show the accuracy and efficiency of the scaled boundary finite element method applied to laminated plate bending problems.

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Contents

1. Introduction . . . 263
2. Virtual work balance . . . 264
3. Scaled boundary finite element formulation . . . 265
4. Solution procedure . . . 266
5. Determination of singularity orders of a symmetric cross-ply laminate . . . 267
6. Conclusion . . . 269
References . . . 270

1. Introduction

The scaled boundary finite element method (SBFEM) is a semi-analytical analysis technique, which combines advantages of the finite element method (FEM) and the boundary element method (BEM). In contrast to the boundary element method, no fundamental solution is needed. The beginnings of the SBFEM are in soil mechanics. In 1982, Dasgupta [1] presented a finite element formulation for the computation of unbounded homogeneous media, which he called cloning algorithm. This method was enhanced by Wolf and Song for the computation of the dynamic stiffnesses of unbounded media [2–4]. The first papers, dealing with the SBFEM, are dedicated to problems of soil mechanics. An overview about the diverse works of Song and Wolf is given in [2]. The mechanical behaviour of a basement to its surrounding soil is investigated by Deeks and Wolf [5]. Doherty and Deeks present an axis-symmetric formulation of the method, where a basement is described and the stiffness of the underlying ground increases with the depth [6,7]. But the SBFEM is not restricted to soil mechanics.

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Deeks and Wolf [8] present a formulation of the method via the virtual work principle, which is aligned to the traditional virtual work derivation of the standard finite element method.

Essential to the SBFEM is, that only the boundary is discretised in a finite element manner. The interesting domain is described by the mapping of its boundary with respect to a scaling centre. This is achieved, introducing the so-called scaled boundary coordinates. A scaling coordinate ξ runs from the scaling centre to the boundary, whereas the boundary coordinate η describes the circumferential direction along the boundary. The displacements are approximated as products of displacement shape functions and unknown functions of the scaling coordinate ξ. Via the virtual work principle, a system of ordinary differential equations for the calculation of the unknown functions is obtained, which can be solved in a closed-form analytical manner. The displacements are expanded in form of a power series. This enables the robust calculation of intensities and exponents of singular fields without any additional effort, if the scaling centre is put in the crack tip, making the SBFEM especially useful for the analysis of cracked and notched structures. The evaluation of stress singularities is not a new topic, a detailed overview is given by Sinclair [9,10]. The benefit of the SBFEM is, that the method combines the versatility and applicability of the FEM with the accuracy of analytical methods. So a wide range of applications, where stress singularities occur, can be analysed with the SBFEM.

One of the first works within the SBFEM dealing with stress singularities in an orthotropic plate under shear is a publication of Song [11]. In another work, Song evaluates power law singularities at cracks and multi-material corners [12]. Mittelstedt and Becker investigate the stress singularities at the free edges of composite laminates using the SBFEM [13,14]. This work is continued by Lindemann and Becker [15,16] and later by Artel and Becker [17]. Müller et al. use the SBFEM for the prediction of the direction of an emanating crack [18]. Song and Vrcelj apply the SBFEM for the calculation of dynamic stress intensity factors [19]. Yang couples the SBFEM with the FEM for simulating cohesive crack growth [20,21]. Mayland and Becker [22] and Li et al. [23] present a formulation of the SBFEM for the analysis of stress singularities in piezoelectric multi-material systems. Finally, Li et al. [24] investigate the influence of various electric boundary conditions and different crack media on the stress intensity factors in piezoelectric bimaterial interfaces.

Besides this, different works dealing with the SBFEM are dedicated to the improvement of the convergence behaviour of the SBFEM. Song introduces matrix-functions for solving the ODE-system [25] and Li et al. use the Schur decomposition for the evaluation of the eigenvalues [26]. Even though the SBFEM has been applied to many problems of continuum mechanics with great success, the application of the SBFEM to plate bending problems is rather unexploited. In 2011, Dieringer et al. [27] published a new formulation of the SBFEM for plate bending problems in the framework of Kirchhoff's plate bending theory. Man et al. [28,29] presented a unified three-dimensional technique, which allows the analysis of plate bending problems with the SBFEM. But in difference to [27] no kinematic assumptions of Kirchhoff's plate theory are enforced.

The present study provides the employment of the SBFEM to problems within the framework of classical laminated plate theory. The first part of the paper is dedicated to the derivation of the scaled boundary finite element equations in displacements for composites. In the second part, stress singularities in a notched composite are examined. The first work dealing with the evaluation of singularities in Kirchhoff plates was published by Williams in 1951 [30]. Williams developed the Eigenfunction method for the evaluation of stress singularities in Kirchhoff plates [30,31]. The method was enhanced by Sinclair for the determination of logarithmic stress singularities [32]. Sih and Rice investigate singularities at bimaterial interfaces in Kirchhoff plates [33]. Labossiere and Huang [34] deal with the examination of singularities in notched plates using Reissner's kinematics. They compare Kirchhoff and Reissner kinematics. Rössle and Sändig present the singularities in notched plates as a function of the notch opening angle using Reissner's plate bending theory for a wide variety of different boundary conditions on the crack faces [35].

As mentioned before, the SBFEM is a semi-analytical analysis technique. The formulation of the SBFEM in the present work is similar to the Kantorovich method. In contrast to the extended Kantorovich method, where a solution is assumed as a sum of products of functions in one direction and functions in the other direction, which are both a priori unknown, only the functions of the scaling coordinate are unknown in the SBFEM and have to be identified. Similar to the SBFEM, the extended Kantorovich method after reduction leads to a system of ordinary differential equations in one direction. Using the Kantorovich method, this system is solved in one direction and then the solutions are used as assumed functions to solve the problem in the other direction. This procedure is repeated iteratively until convergence is completed. A lot of researchers use the extended Kantorovich method for example for analysing the free-edge strength of composite laminates [36], the bending of thick laminated plates [37] and the large deflection of laminated rectangular plates under general out-of-plane loading [38]. Shufrin et al. investigated the buckling of symmetrically laminated plates using the Kantorovich method, they discussed the convergence behaviour and pointed out that the solution is independent of the initial selected functions, even if a selected function does not satisfy the boundary conditions [39]. Other numerical methods for analysing composites are the finite strip method [40] and the finite element method [41,42]. Beside these methods, there are different meshless methods for the analysis of plates. Krysl and Belytschko [43] were the first, who applied the element-free Galerkin method to the static analysis of thin plates. An overview about the development of element-free or meshless methods and their applications for the analysis of composite structures is given by Liew et al. [44].

The enhancement of the SBFEM in the present work enables the evaluation of stress singularities as a function of the notch opening angle for a wide variety of composites. The influences of boundary conditions at the crack faces on the singularities are also investigated, power logarithmic stress singularites are detected in a proper way and are also presented. Couplings between in-plane and out-of-plane behaviour of the composites as well as their influence on the stress singularities are also rendered. Numerical examples demonstrate the accuracy and efficiency of the method's application to arbitrarily laminated plates.

2. Virtual work balance

If sufficiently thin laminated plates are examined, shear deformations can be neglected and the kinematical assumptions of Kirchhoff hold. The virtual work balance for a composite laminate of domain Ω and boundary Γ can be expressed by the following relation, if hygrothermal and body loads are not taken into account:

$$\int_\Omega \left[(\mathcal{L}\delta\mathbf{u}_0)^T\mathbf{D}_A\mathcal{L}\mathbf{u}_0 - (\mathcal{L}\delta\mathbf{u}_0)^T\mathbf{D}_B\mathcal{L}\nabla w - (\mathcal{L}\nabla w)^T\mathbf{D}_B\mathcal{L}\mathbf{u}_0 + (\mathcal{L}\nabla w)^T\mathbf{D}_D\mathcal{L}\nabla w\right]\mathrm{d}\Omega$$
$$= \int_\Gamma \left[\delta u_n\overline{N}_n + \delta u_t\overline{N}_t - \delta w_{,n}\overline{M}_n - \delta w_{,t}\overline{M}_{nt} + \delta w\overline{S}_n\right]\mathrm{d}\Gamma.$$

The in-plane displacements are denoted by $\mathbf{u}$, whereas the out-of-plane displacement is given by w. The differential operator $\mathcal{L}$ relates the strains and the displacements. The quantities $\mathbf{D}_i$ represent material matrices. The symbols with overbars denote stress

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