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Stacking sequence optimisation of variable stiffness laminates with manufacturing constraints

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ABSTRACT

The fibre paths of variable stiffness laminates are described through the fibre angles at the nodes of a finite element (FE) representation of the structure. An algorithm is presented to optimise the fibre angles efficiently. To reduce the number of required FE analyses a multi-level approach is used: the exact solution is first approximated in laminate stiffness space. The second level approximation is a Gauss–Newton quadratic approximation in fibre angle space. To ensure manufacturability, a steering constraint is introduced: the norm of the gradient of the fibre angle distribution is constrained. Two formulations are proposed: either the average steering is constrained; or the local element-wise steering is constrained. The resulting quadratically constrained quadratic optimisation problem is solved using an interior-point method. It is shown that the local steering constraint performs best, at the cost of increasing the size of the problem.

1. Introduction

Today, composite materials are frequently used in the aviation industry and the first composite-dominated planes like the B-787 or A400M are being built. Traditionally, fibres within a layer have the same orientation, leading to constant stiffness properties. However, as manufacturing technology evolved, for example automated fibre placement machines, the fibre orientation of a layer can be continuously varied leading to varying stiffness properties that are best tailored for the applied loads. These composites are called variable stiffness laminates (VSL).

When designing VSL, manufacturability is not always taken into account [1]. In an early work, the structure was divided into different segments in which the fibre angle was optimised separately. An example of an outcome can be seen in Fig. 1(a) [2]. A similar approach has been taken in the optimisation of flutter speed for wings: the angle of a lot of elements is optimised, but no manufacturability constraint is taken into account [3]. In another approach, the change in fibre angle between adjacent layers is taken into account, but the set of possible angles is restricted to 0°, ±45° and 90°, so the change in fibre angle is still large [4].

To take manufacturability into account, linearly varying fibre angles were used as can be seen in Fig. 1(b), which has given promising, manufacturable, results [5–10]. Also for stiffened plates, the use of linearly varying fibre angle per bay has been investigated, and again it was shown that varying the fibre angles leads to better performance [11,12]. Direct parametrisation of the tow paths using Lagrangian polynomials, splines or NURBS (Non-Uniform Rational B-Splines) has been done as well. This also showed large, manufacturable, improvements in buckling load, but the result is dependant of the basis functions you chose to incorporate [13–15]. Hence, the total potential of VSL is not exploited due to the pre-specified set of possibilities. Furthermore, most methods assume the fibres are shifted, meaning a choice had to be made whether gaps or overlaps were allowed during manufacturing [16].

Another approach that leads to manufacturable designs is to align the fibres in the direction of principal stress. This was shown to reduce stress concentrations, and could also lead to reduced weight using the tailored fibre placement method [17,18]. Also using the load paths, or a hybrid combination of load paths and principal stress direction has been used to design variable stiffness laminates [19]. Continuous tow shearing is a new manufacturing method, leading to varying fibre angles without any gaps or overlaps, but with a thickness variation that is coupled with the change in fibre angle [20,21]. Using a genetic algorithm, coupled with a pattern-search algorithm, or using the infinite strip method large improvements were shown to be possible [22,23]. A more comprehensive review can be found in Ghiasi et al. [24].

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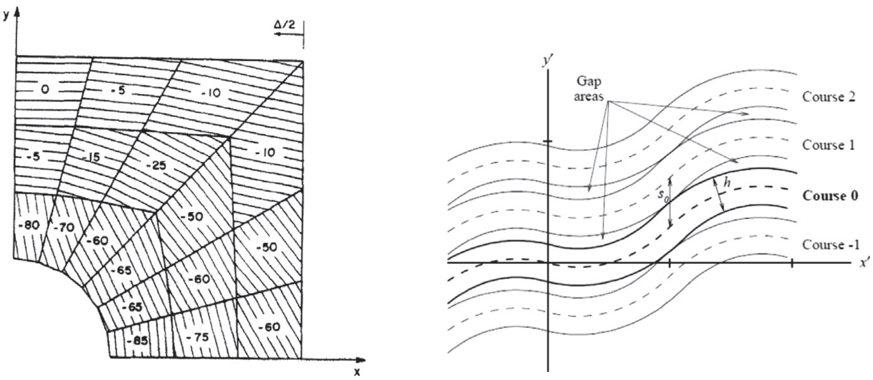

(a) example of the outcome when structure is divided in parts, taken from Hyer and Lee [2]

(b) example of a linearly varying fibre angle, taken from Lopes et. al. [5]

Fig. 1. 2 Outcomes of previous optimisations.

To exploit the possibilities of VSL fully, a three-step approach has been developed. The first step is to find the optimal stiffness distribution in terms of the lamination parameters. This is discussed in detail in IJsselmuiden [25,26]. The second step is to find the optimal manufacturable fibre angle distribution, the focus of this paper [27–29]. The third step is to retrieve the fibre paths, discussed in Blom [30]. A schematic overview of this approach is shown in Fig. 2.

In this paper an algorithm to optimise manufacturable fibre angle distributions is developed. The stacking sequence at each node of a finite element model will be optimised. To ensure manufacturability, the rate of change in fibre angle between nodes, referred to as steering from here on, is constrained [31,32].

This paper is organised as follows: first the problem formulation is discussed in Section 2, next the manufacturing constraints are discussed in Section 3. The solution procedure is explained in Section 4, followed by the results in Section 5, and finally the conclusion in Section 6.

2. Problem formulation

In structural optimisation, the minimisation of an objective (e.g., weight or compliance) subject to performance constraints (e.g., on stresses) is studied. More generally, the worst case response (e.g., in the case of multiple load cases) may be minimised. Additional constraints, often arising from manufacturing considerations, may be imposed to guarantee certain properties of the design.

In the optimisation of variable stiffness laminates a suitably-defined steering norm ς is constrained to be less than a maximal steering value ς_U representing the upper manufacturing limit. More details about the formulation of the steering constraint will be given in Section 3.

Thus, the following general problem formulation is considered:

$$\begin{aligned} \min \quad & \max(f_1, f_2, \ldots, f_n) \\ s.t. \quad & f_{n+1}, \ldots, f_m \leqslant 0 \\ & \varsigma^2 - \varsigma_U^2 \leqslant 0 \end{aligned} \tag{1}$$

where f_1 up to f_n denote structural responses that are optimised and f_{n+1} up to f_m are constraints.

Structural responses, such as stiffness and strength, are calculated using finite element analyses (FEA). Since each FEA is computationally expensive, greater efficiency may be achieved by using structural approximations, reducing the number of FEAs [33]. The exact FE response f is first approximated in terms of the in- and out-of-plane stiffness matrices $\boldsymbol{A}$ and $\boldsymbol{D}$ and their reciprocals:

$$f^{(1)} \approx \sum_n \boldsymbol{\phi}_m : \boldsymbol{A}^{-1} + \boldsymbol{\phi}_b : \boldsymbol{D}^{-1} + \boldsymbol{\psi}_m : \boldsymbol{A} + \boldsymbol{\psi}_b : \boldsymbol{D} + c \tag{2}$$

where the : operator represents the Frobenius inner product, meaning $\boldsymbol{A} : \boldsymbol{B} = tr(\boldsymbol{A} \cdot \boldsymbol{B}^T)$; the reciprocal and linear approximation terms $\boldsymbol{\phi}$ and $\boldsymbol{\psi}$ are calculated from a sensitivity analysis [34,35], m denotes the membrane, b the bending part and n runs over all the nodes. This approximation is a generalisation of the linear-reciprocal ones used in the convex linearisation method [36]. The approximations are convex in stiffness space provided that $\boldsymbol{\phi} \geqslant 0$, which is always satisfied by construction. For many responses that enjoy homogeneity properties the free term c equals zero. This way of approximating structural responses works for stiffness, buckling, strength and eigenfrequency problems.

To optimise the fibre angles, the first approximation is evaluated by considering the dependence of the stiffness matrices on the fibre angles. As a function of fibre angles, it no longer has a simple mathematical form and is not generally convex. Hence, a second level approximation is made:

$$f^{(2)} \approx f_0^{(1)} + \boldsymbol{g} \cdot \Delta\boldsymbol{\theta} + \Delta\boldsymbol{\theta}^T \cdot \boldsymbol{H} \cdot \Delta\boldsymbol{\theta} \tag{3}$$

where $f_0^{(1)}$ denotes the value, $\boldsymbol{g}$ the gradient and $\boldsymbol{H}$ is an approximation of the Hessian of the first approximation at the approximation point. $\boldsymbol{g}$ and $\boldsymbol{H}$ can be calculated starting from

$$f^{(2)}(\theta) = f^{(1)}(\boldsymbol{s}(\theta)) \tag{4}$$

where $\boldsymbol{s}$ contains the components of the stiffness matrices $\boldsymbol{A}$ and $\boldsymbol{D}$. Deriving this leads to

$$g_i = \frac{\partial f^{(1)}}{\partial \theta_i} = \frac{\partial f^{(2)}}{\partial \theta_i} = \frac{\partial f^{(1)}}{\partial \boldsymbol{s}_\alpha} \cdot \frac{\partial \boldsymbol{s}_\alpha}{\partial \theta_i} \tag{5}$$

Deriving again, the Hessian is found to be

$$H_{ij} = \frac{\partial^2 f^{(1)}}{\partial \theta_i \partial \theta_j} = \frac{\partial^2 f^{(1)}}{\partial \boldsymbol{s}_\alpha \partial \boldsymbol{s}_\beta} \cdot \frac{\partial \boldsymbol{s}_\alpha}{\partial \theta_i} \cdot \frac{\partial \boldsymbol{s}_\beta}{\partial \theta_j} + \underline{\frac{\partial f^{(1)}}{\partial \boldsymbol{s}_\alpha} \cdot \frac{\partial^2 \boldsymbol{s}_\alpha}{\partial \theta_i \partial \theta_j}} \tag{6}$$

Convexity is guaranteed by omitting the underlined part of Eq. (6), which is not guaranteed to be positive definite, and using the Gauss–Newton part which is positive semi-definite. An approximation has to have equal function and gradient at the approximation point as the approximated function. Hence, using only part of the Hessian does give a valid approximation.