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Extension-shear coupled laminates with immunity to hygro-thermal shearing distortion

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ABSTRACT

Material-independent necessary and sufficient conditions are derived for extension-shear coupled laminate with immunity to hygro-thermal shearing distortion (HTSD) using classical lamination theory. It is proven that no standard form of extension-shear coupled laminate with immunity to HTSD exists. A survey is performed to identify free form of extension-shear coupled laminate with immunity to HTSD that produce maximum extension-shear coupling. This is achieved through a constrained optimization routine, where the constraints are the material-independent necessary and sufficient conditions for laminates with immunity to HTSD. Results are presented for the symmetric laminates that consist of 8–14 plies. Comparisons are made on hygrothermal behavior and buckling strength of extension-shear coupled laminates with and with immunity to HTSD. The robustness of the extension-shear coupled laminates with immunity to HTSD is verified by using Monte Carlo simulation for laminates with fiber orientation that slightly deviates from the theoretically designed direction.

1. Introduction

Composite laminates fabricated from identical orthotropic plies can be tailored by arranging each ply's fiber orientation angle to produce the desired couplings [1]. However, direct application of the laminates is not ideal for most purposes, and it's usually necessary to introduce the laminates into structure configurations with desired coupling. For example, a tilt-rotor blade with extension-twist coupling can acquire an optimal twist distribution along the blade to achieve an angle of attack in both the vertical and forward flight regimes. Extension-twist coupling turbine blades may serve as a passive load-alleviation mechanism during extreme wind conditions, and where failure of an active control system may lead to destruction of the entire wind turbine [2]. The bend-twist coupled wing in swept-forward-wing aircraft could delay the onset of aeroelastic instabilities.

Extension-shear coupled laminate, as shown in Fig. 1, has been employed extensively to achieve extension-twist coupling or bend-twist coupling at the structural level. Through off-axis alignment of a balanced and symmetric laminate with extension-shear coupling, Nixon [3] designed a tilt-rotor blade with extension-twist coupling. However, such laminates possess significant bend-twist coupling at the laminate level, leading to detrimental effects on the compression buckling strength of the blade [4]. Baker [5] achieved bend-twist behavior at the wing-box level through un-balanced and symmetric laminates with extension-shear coupling. Bend-twist coupling at the laminate level is again present in such designs. York [6] derived a definitive list of un-balanced and non-symmetric laminates only with extension-shear coupling, which can be manufactured flat under a standard elevated temperature curing process. York also proved that compression buckling strength can be substantially improved in the absence of laminate level bend-twist coupling without affecting the bending-twisting response at the wing-box level. Using hygro-thermal stability at the laminate level as constraints, Muder [7] developed optimal laminates for extension-shear coupling to design closed cell configuration with extension-twist coupling.

As mentioned above, studies on extension-shear coupled laminates only focus on hygro-thermal curvature stability at the laminate level. It insures the laminate is immune to thermal warping distortion which occurs on cooling after the elevated temperature curing process used in the manufacture [8]. However, it is noted that the hygro-thermally curvature-stable laminates do not guarantee hygro-thermal curvature stability when the laminates are introduced into structure configuration. For example, when the extension-shear coupled laminates are introduced into wing box configuration, the concordant HTSD at the laminate level with respect to the mid plane could induce bending distortion in box-beam configuration, as shown in Fig. 2. The laminates in the

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forward and aft spars of the box-beam are isotropic, while the bias angles in the top and bottom skins are in the same direction to achieve bending-twisting coupling in wing box configuration.

The aim of the current paper is to identify the extension-shear coupled laminates with immunity to HTSD, beyond the limited scope of that currently available in the literature. This work begins with a derivation of material-independent necessary and sufficient conditions for extension-shear coupled laminate with immunity to hygrothermally induced shearing distortion using classical lamination theory (CLT). Using these conditions as constraints, families of extension-shear coupled laminates with immunity to HTSD are derived through a constrained optimization routine. Comparisons are then made between hygrothermal behavior and buckling strength of extension-shear coupled laminates with and with immunity to HTSD. A sensitivity analysis of the errors in layup is further made to indicate the robustness of the extension-shear coupled laminates with immunity to HTSD.

2. Development of necessary and sufficient conditions

For simplicity, only thermal effect is considered in the subsequent analysis. Using CLT, a system of linear equations describing the constitutive relationships read

$$\begin{Bmatrix} N_x^T \\ N_y^T \\ N_{xy}^T \\ M_x^T \\ M_y^T \\ M_{xy}^T \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^T \\ \varepsilon_y^T \\ \gamma_{xy}^T \\ \kappa_x^T \\ \kappa_y^T \\ \kappa_{xy}^T \end{Bmatrix} \tag{1}$$

where $[\mathbf{N}^T]$ and $[\mathbf{M}^T]$ are the thermal stress resultants and thermal moment, $[\boldsymbol{\varepsilon}^T]$ and $[\boldsymbol{\kappa}^T]$ are the in-plane strains and curvatures in a laminated plate. Elements of the stiffness matrices are related to the lamination parameters and laminate invariants by

$$\begin{Bmatrix} A_{11} \\ A_{12} \\ A_{22} \\ A_{66} \\ A_{16} \\ A_{26} \end{Bmatrix} = H \begin{bmatrix} \mathbf{1} & \xi_1 & \xi_2 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & -\xi_{bf2} & \mathbf{1} & \mathbf{0} \\ \mathbf{1} & -\xi_1 & \xi_2 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & -\xi_2 & \mathbf{0} & \mathbf{1} \\ \mathbf{0} & \frac{\xi_3}{2} & \xi_4 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \frac{\xi_3}{2} & -\xi_4 & \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{Bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \end{Bmatrix} \tag{2}$$

$$\begin{Bmatrix} B_{11} \\ B_{12} \\ B_{22} \\ B_{66} \\ B_{16} \\ B_{26} \end{Bmatrix} = \frac{H^2}{4} \begin{bmatrix} \mathbf{0} & \xi_5 & \xi_6 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & -\xi_6 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & -\xi_5 & \xi_6 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & -\xi_6 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \frac{\xi_7}{2} & \xi_8 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \frac{\xi_7}{2} & -\xi_8 & \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{Bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \end{Bmatrix} \tag{3}$$

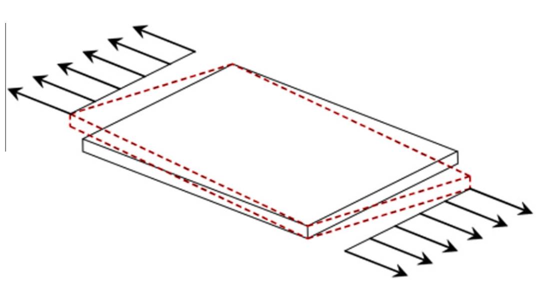

Fig. 1. Extension-shear coupling.

$$\begin{Bmatrix} D_{11} \\ D_{12} \\ D_{22} \\ D_{66} \\ D_{16} \\ D_{26} \end{Bmatrix} = \frac{H^3}{12} \begin{bmatrix} \mathbf{1} & \xi_9 & \xi_{10} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & -\xi_{10} & \mathbf{1} & \mathbf{0} \\ \mathbf{1} & -\xi_9 & \xi_{10} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & -\xi_{10} & \mathbf{0} & \mathbf{1} \\ \mathbf{0} & \frac{\xi_{11}}{2} & \xi_{12} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \frac{\xi_{11}}{2} & -\xi_{12} & \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{Bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \end{Bmatrix} \tag{4}$$

where

$$\begin{aligned} (\xi_1 \;\; \xi_2 \;\; \xi_3 \;\; \xi_4) &= \tfrac{1}{2}\sum_{k=\mathbf{1}}^{n} (\cos\mathbf{2}\theta_k \;\; \cos\mathbf{4}\theta_k \;\; \sin\mathbf{2}\theta_k \;\; \sin\mathbf{4}\theta_k)\left(\tfrac{2z_k}{H} - \tfrac{2z_{k-1}}{H}\right) \\ (\xi_5 \;\; \xi_6 \;\; \xi_7 \;\; \xi_8) &= \tfrac{1}{2}\sum_{k=\mathbf{1}}^{n} (\cos\mathbf{2}\theta_k \;\; \cos\mathbf{4}\theta_k \;\; \sin\mathbf{2}\theta_k \;\; \sin\mathbf{4}\theta_k)\left[\left(\tfrac{2z_k}{H}\right)^2 - \left(\tfrac{2z_{k-1}}{H}\right)^2\right] \\ (\xi_9 \;\; \xi_{10} \;\; \xi_{11} \;\; \xi_{12}) &= \tfrac{1}{2}\sum_{k=\mathbf{1}}^{n} (\cos\mathbf{2}\theta_k \;\; \cos\mathbf{4}\theta_k \;\; \sin\mathbf{2}\theta_k \;\; \sin\mathbf{4}\theta_k)\left[\left(\tfrac{2z_k}{H}\right)^3 - \left(\tfrac{2z_{k-1}}{H}\right)^3\right] \end{aligned} \tag{5}$$

$$\begin{aligned} &U_1 = \{\mathbf{3}Q_{11} + \mathbf{3}Q_{22} + \mathbf{2}Q_{12} + \mathbf{4}Q_{66}\}/\mathbf{8} \quad U_2 = \{Q_{11} - Q_{22}\}/\mathbf{2} \\ &U_3 = \{Q_{11} + Q_{22} - \mathbf{2}Q_{12} - \mathbf{4}Q_{66}\}/\mathbf{8} \\ &U_4 = \{Q_{11} + Q_{22} + 6Q_{12} - \mathbf{4}Q_{66}\}/\mathbf{8} \quad U_5 = \{Q_{11} + Q_{22} - 2Q_{12} + Q_{66}\}/\mathbf{8} \end{aligned} \tag{6}$$

in which θ_k and z_k represent the fiber orientation angle of the kth ply and the distance from the kth ply to mid-plane, respectively.

Laminates possessing extension-shear coupling only are referred to by the designation $\mathbf{A}_F\mathbf{B}_0\mathbf{D}_S$, signifying that all elements of the matrix $[\mathbf{A}]$ are finite, i.e. $A_{ij} \neq 0$, the matrix $[\mathbf{B}]$ is null, and the elements of the matrix $[\mathbf{D}]$ are specially orthotropic in nature, i.e. $D_{16} = D_{26} = 0$[4]. For $\mathbf{A}_F\mathbf{B}_0\mathbf{D}_S$ laminate, Eq. (1) can be simplified to give

$$\begin{Bmatrix} N_x^T \\ N_y^T \\ N_{xy}^T \\ M_x^T \\ M_y^T \\ M_{xy}^T \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ A_{12} & A_{22} & A_{26} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ A_{16} & A_{26} & A_{66} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & D_{11} & D_{12} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & D_{12} & D_{22} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & D_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^T \\ \varepsilon_y^T \\ \gamma_{xy}^T \\ \kappa_x^T \\ \kappa_y^T \\ \kappa_{xy}^T \end{Bmatrix} \tag{7}$$

With Eq. (7), the constitutive relationships between the thermal stress resultants $[\mathbf{N}^T]$ and in-plane strains $[\boldsymbol{\varepsilon}^T]$ in a laminated plate read

$$\begin{Bmatrix} N_x^T \\ N_y^T \\ N_{xy}^T \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^T \\ \varepsilon_y^T \\ \gamma_{xy}^T \end{Bmatrix} \tag{8}$$

Thus, the in-plane thermal strains can be calculated through multiplying the inverse of the $[\mathbf{A}]$ matrix by thermal stress resultants $[\mathbf{N}^T]$

$$\begin{Bmatrix} \varepsilon_x^T \\ \varepsilon_y^T \\ \gamma_{xy}^T \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix}^{-1} \begin{Bmatrix} N_x^T \\ N_y^T \\ N_{xy}^T \end{Bmatrix} = \frac{A^*}{|A|} \begin{Bmatrix} N_x^T \\ N_y^T \\ N_{xy}^T \end{Bmatrix} \tag{9}$$

in which

$$A^* = \begin{bmatrix} A_{22}A_{66} - A_{26}^2 & A_{16}A_{26} - A_{12}A_{66} & A_{12}A_{26} - A_{16}A_{22} \\ A_{16}A_{26} - A_{12}A_{66} & A_{11}A_{66} - A_{16}^2 & A_{12}A_{16} - A_{11}A_{26} \\ A_{12}A_{26} - A_{16}A_{22} & A_{12}A_{16} - A_{11}A_{26} & A_{11}A_{22} - A_{12}^2 \end{bmatrix} \tag{10}$$

With Eqs. (9) and (10), the thermally induced shear deformation, γ_{xy}^T, can be obtained as

$$\gamma_{xy}^T = \frac{\mathbf{1}}{|A|}\left[(A_{12}A_{26} - A_{16}A_{22})N_x^T + (A_{12}A_{16} - A_{11}A_{26})N_y^T + (A_{11}A_{22} - A_{12}^2)N_{xy}^T\right] \tag{11}$$

The thermal force resultants are related to the lamination parameters and thermal invariants by