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A simple homogenization scheme for 3D finite element analysis of composite bolted joints

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Usually a three-dimensional (3D) finite element model is used for analyzing laminated composite bolted joints, because the stress and strain vary in all directions due to factors such as bolt bending and tilting, bolt pre-load and secondary bending. In general, the advantage of symmetry can greatly reduce the modeling and computational time in a 3D analysis. However, due to different lamination angles in laminated composite structures, symmetric model may not always be possible in most of the cases. To overcome this problem, an equivalent homogeneous 3D elasticity model is developed and presented in this paper for simplified analysis of laminated composite bolted joints. This type of model is beneficial for determining the basic mechanical behaviors of a laminated composite joint with significantly lesser computational effort. The present formulation is based on Mindlin–Reissner plate theory which takes into account the effect of transverse shear deformation in composite laminates. The present method is capable of finding equivalent homogeneous model for the laminated composite joints having any type of lamination scheme. This homogenization scheme is also useful for developing simplified analytical models (e.g., spring based models) for laminated composite bolted joints.

1. Introduction

Joints in laminated composite structures play very significant role on overall performance of the whole structure. Failure of the structures often initiates from the location of joints. Therefore, understanding the complete behavior of FRP joints by means of proper analysis is important for the successful performance of FRP laminated structures. The bolted joints are difficult to analyze because the stresses as well as strains vary three dimensionally due to factors such as secondary bending, bolt bending, tilting and bolt pre-load. Therefore, 3D solid modeling is essential to completely understand the behavior of the laminated composite bolted joints. Due to arbitrary fiber orientations in different layers of a composite laminate, one cannot always use symmetric model for the laminated composite bolted joints to minimize the modeling and computational time. To overcome this problem, an equivalent homogeneous model will be quite useful to understand the overall behavior of the laminated composite bolted joints. This type of simplified model can predict the mechanical behaviors such as: joint stiffness, surface strains, displacements and average through-thickness stresses with sufficient accuracy.

Many experimental, analytical and numerical studies are available in literatures for understanding the behavior of laminated composite bolted joints. Lawlor et al. [1] did some experimental investigations to understand the structural behavior of a single-lap and single-bolt composite joints. Lawlor et al. [2] performed an experimental investigation on multi-bolt, double-lap composite joints to study the load distribution, strength, failure modes and fatigue life. Nelson et al. [3] presented a spring-based model for single-lap composite joints. McCarthy and Gray [4] developed an analytical model for shear loading in single or multi-bolt composite joints, taking effects of varying bolt-torque and bolt-hole clearance. Gray and McCarthy [5] modified the spring-based analytical model with Ritz approximation model for considering the through-thickness stiffness of single-bolt and single-lap composite joints subjected to out of plane loading. But, only three-dimensional models can provide a detailed simulation of the effects occur in the vicinity of bolt-laminate. McCarthy et al. [6] developed a three dimensional finite element model to study the mechanical behavior of a single-bolt and single-lap composite joint. McCarthy and McCarthy [7] further studied the effects of clearance on joint stiffness, stress state and failure initiation. McCarthy et al. [8] presented a three dimensional finite element progressive damage model of multi-bolt and double-lap composite joints using ABAQUS 6.12 [9].

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Homogeneous models are already popular for the sandwich structures with various core configurations. Aref et al. [10] transformed a rib core sandwich panel to a homogeneous, orthotropic Kirchhoff plate layer with the same global stiffness. Some researchers investigated the equivalent homogeneous model for the sandwich panels with truss-core [11,12], corrugated core [13,14] and Z-core [15]. The transverse shear stiffness for various core configurations has been calculated analytically and numerically by Nordstrand et al. [16]. The equivalent property only for the core portion of corrugated sandwich plate have been investigated by Bartolozzi et al. [17] and Bartolozzi et al. [18]. But, homogeneous model for laminated composite thick plate based on 3D elasticity which will be useful for FRP bolted joint, is rare in literature.

A method for finding equivalent homogeneous property is presented in this paper considering 3D elasticity for laminated composite plates used in bolted joints. To homogenize the laminated plate with arbitrarily oriented layers (i.e., non-homogeneous), the equivalent stiffness matrix is calculated considering bending, twisting and shear stiffnesses. The Mindlin–Reissner plate theory which includes the transverse shear deformations of plate has been used in the formulation of classical laminate theory. A MATLAB program has been developed for this purpose. This homogeneous model is useful for analyzing any type of bolted joints made of thin to thick composite laminate and it also saves significant computational time. Moreover, the advantage of symmetry can also be exploited in the FE modeling by using the proposed homogenized plate model. The present method is capable of finding equivalent homogeneous orthotropic engineering constants for symmetric laminates having any type of lamination scheme. When the laminate is not symmetric, the present method can be used to generate equivalent stiffness matrix, containing coupling between axial and bending stiffness. The present homogenization model has been validated with some published results and also by developing an actual 3D layered solid FE model considering various lamination schemes. The present 3D FE modeling and analysis of the bolted joints have been carried out using finite element package ABAQUS.

2. Mathematical formulation

Fasteners (bolts/nuts/washers) used in laminated composite bolted joints are usually made of homogeneous and isotropic materials. In the present formulation the equivalent stiffness matrix is calculated for inhomogeneous, anisotropic laminated plates of composite bolted joints, considering bending, twisting and shear stiffnesses. The Mindlin–Reissner plate theory which takes into account transverse shear deformations through-the-thickness of plate is used in the formulation of classical laminate theory.

2.1. Equivalent plate stiffness

If $[S_{ij}]_k$ is the elastic compliance matrix for the kth lamina of the laminated plate then, orthotropic stress–strain relationship for the lamina from 3D elasticity can be written as:

$$\{\varepsilon_1 \quad \varepsilon_2 \quad \varepsilon_3 \quad \gamma_{12} \quad \gamma_{13} \quad \gamma_{23}\}_k^T = [S_{ij}]_k\{\sigma_1 \quad \sigma_2 \quad \sigma_3 \quad \sigma_6 \quad \sigma_4 \quad \sigma_5\}_k^T \tag{1}$$

i.e. $\{\varepsilon\}_k = [S_{ij}]_k\{\sigma\}_k$

$$\text{where } [S_{ij}]_k = \begin{bmatrix} E_1^{-1} & -\nu_{21}E_2^{-1} & -\nu_{31}E_3^{-1} & 0 & 0 & 0 \\ -\nu_{12}E_1^{-1} & E_2^{-1} & -\nu_{32}E_3^{-1} & 0 & 0 & 0 \\ -\nu_{13}E_1^{-1} & -\nu_{23}E_2^{-1} & E_3^{-1} & 0 & 0 & 0 \\ 0 & 0 & 0 & G_{12}^{-1} & 0 & 0 \\ 0 & 0 & 0 & 0 & G_{13}^{-1} & 0 \\ 0 & 0 & 0 & 0 & 0 & G_{23}^{-1} \end{bmatrix}_k \tag{2}$$

$$\text{So,} \quad \{\sigma\}_k = [S_{ij}]_k^{-1}\{\varepsilon\}_k \quad \text{or} \quad \{\sigma\}_k = [C_{ij}]_k\{\varepsilon\}_k \tag{3}$$

where $[C_{ij}]_k$ is the elastic constitutive matrix of kth lamina with respect to the material axis system. If θ is the lamination angle of the kth lamina with respect to global x axis then with respect to global axis system $[C_{ij}]_k$ can be represented as:

$$[C_{xy}]_k = [T]^{-1} \times [C_{ij}]_k \times [T]^{-T} \tag{4}$$

where transformation matrix $[T]$ is given by

$$[T] = \begin{bmatrix} m^2 & n^2 & 0 & 2mn & 0 & 0 \\ n^2 & m^2 & 0 & -2mn & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ -mn & mn & 0 & m^2-n^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & m & n \\ 0 & 0 & 0 & 0 & -n & m \end{bmatrix} \tag{5}$$

where $m = \cos\theta$ and $n = \sin\theta$. The detailed derivation of the transformation matrix is given in Appendix A.

The final transformed and combined stiffness matrix of the plate is calculated at the mid plane of the structure. For that, resultant forces and moments due to each lamina are calculated by integrating them across the thickness of each lamina k and summing up the forces for all lamina of the laminate. Force resultants for the plate can be written as:

$$\begin{Bmatrix} N_x \\ N_y \\ N_z \\ N_s \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{16} \\ A_{21} & A_{22} & A_{23} & A_{26} \\ A_{31} & A_{32} & A_{33} & A_{36} \\ A_{61} & A_{62} & A_{63} & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \varepsilon_z^0 \\ \gamma_s^0 \end{Bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{13} & B_{16} \\ B_{21} & B_{22} & B_{23} & B_{26} \\ B_{31} & B_{32} & B_{33} & B_{36} \\ B_{61} & B_{62} & B_{63} & B_{66} \end{bmatrix} \begin{Bmatrix} k_x \\ k_y \\ k_z \\ k_s \end{Bmatrix} \tag{6}$$

i.e. $\boldsymbol{N} = \boldsymbol{A} \times \boldsymbol{\varepsilon^0} + \boldsymbol{B} \times \boldsymbol{\kappa}$

where $[A] = \sum_{k=1}^{n}[C_{xy}]_k(h_k - h_{k-1})$ and $[B] = \frac{1}{2}\sum_{k=1}^{n}[C_{xy}]_k\left(h_k^2 - h_{k-1}^2\right)$ for total n number of plies.

Again, considering mid plane of the plate as position of the reference axis, the moment resultants can be written as:

$$\begin{Bmatrix} M_x \\ M_y \\ M_z \\ M_s \end{Bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{13} & B_{16} \\ B_{21} & B_{22} & B_{23} & B_{26} \\ B_{31} & B_{32} & A_{33} & B_{36} \\ B_{61} & B_{62} & B_{63} & B_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \varepsilon_z^0 \\ \gamma_s^0 \end{Bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{13} & D_{16} \\ D_{21} & D_{22} & D_{23} & D_{26} \\ D_{31} & D_{32} & D_{33} & D_{36} \\ D_{61} & D_{62} & D_{63} & D_{66} \end{bmatrix} \begin{Bmatrix} k_x \\ k_y \\ k_z \\ k_s \end{Bmatrix} \tag{7}$$

i.e. $\boldsymbol{M} = \boldsymbol{B} \times \boldsymbol{\varepsilon^0} + \boldsymbol{D} \times \boldsymbol{\kappa}$

where $[D] = \frac{1}{3}\sum_{k=1}^{n}[C_{xy}]_k\left(h_k^3 - h_{k-1}^3\right)$.

The resultant shear forces can also be written as:

$$\begin{Bmatrix} Q_{xz} \\ Q_{yz} \end{Bmatrix} = \begin{bmatrix} A_{xx}^z & A_{xy}^z \\ A_{yx}^z & A_{yy}^z \end{bmatrix} \begin{Bmatrix} \varepsilon_{xz} \\ \varepsilon_{yz} \end{Bmatrix} \tag{8}$$

i.e. $\boldsymbol{Q} = \boldsymbol{A^z} \times \boldsymbol{\varepsilon_{trans}}$

where, $[A^z] = \kappa_s \times \sum_{k=1}^{n}[C_{xy}]_k(h_k - h_{k-1})$ and κ_s denotes the shear correction factor.

So, the all forces and moments can be expressed as:

$$\{\boldsymbol{N} \quad \boldsymbol{M} \quad \boldsymbol{Q}\}^T = [K_c]\{\boldsymbol{\varepsilon^0} \quad \boldsymbol{\kappa} \quad \boldsymbol{\varepsilon_{trans}}\}^T \tag{9}$$

Where, the final combined stiffness matrix $[K_c]$ for the laminate can be written as:

$$[K_c] = \begin{bmatrix} \boldsymbol{A} & \boldsymbol{B} & 0 \\ \boldsymbol{B} & \boldsymbol{D} & 0 \\ 0 & 0 & \boldsymbol{A^z} \end{bmatrix} \tag{10}$$

For symmetric layups, the contribution of only bending stiffness can be considered, because contribution of the $[B]$ matrix will be very less and the $[A]$ matrix can be neglected because, it is very less compared to the $[D]$ matrix. So, after neglecting $[A]$ and $[B]$ matrices, the