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Modelling and testing of the snap-through process of bi-stable cross-ply composites

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ABSTRACT

Bi-stable response of cross-ply composites is modelled when snap processes are trigged by a vertical force and the laminate is supported at four points. The model is based on the Rayleigh–Ritz technique. Non-linear von Kármán strains, non-uniform curvatures and uniform through-the-thickness normal strain are included in the analysis. The analysis has been carried out step by step, by small increments of the applied load. Experiments in four $[0/90]_T$ carbon/epoxy laminates have been carried out for comparison in a test configuration that fulfils the requirements of the model. The multi-event snap-through and intermediate equilibrium positions have been experimentally observed and recorded. The initial snap-through and snap-back load–displacement curves have been experimentally determined and compared to those corresponding to the proposed model.

1. Introduction

Asymmetric cross-ply laminates have residual curvatures at Room Temperature (RT) due to the elevated temperatures experienced during manufacturing [1–4]. These laminates can have two stable shapes at RT: cylindrical state I with a major curvature in x axis and cylindrical state II with an opposite curvature in the orthogonal y axis. One stable shape can change to the other by applying low energy, as it is shown in Fig. 1. This is known as the snap-through phenomenon [5]. In snap-through the specimen is forced to snap from the first to the second equilibrium shape and in snap-back the specimen snaps from the second to the first equilibrium shape. In multi-stability of laminates, the geometry [6] and the negative influence of moisture effects should be taken into account [7–9]. The study of loss of bifurcation has been widely addressed in [9,10].

Bi-stable composite laminates have been proposed as a choice to generate novel morphing structures [11,12]. Hyer et al. [1,2,13–17] predicted the cured shapes of bi-stable laminates at RT developing an approximate theory based on the Rayleigh–Ritz technique assuming strain and displacement fields, and minimising the total potential energy (TPE). They used low order polynomial functions, linear elastic constitutive equations and nonlinear relations of von Kármán strains. The initial four-term model over predicted the magnitude of the curvatures at RT [16], being these values higher than those in models with more coefficients. Afterwards, other models with low order polynomial functions [18–21] have been proposed.

With respect to snap-through, Dano and Hyer [22,23] predicted snap-through forces and moments using the Rayleigh–Ritz technique in conjunction with the principle of virtual work and they developed an actuation solution based on shape memory alloys. Diaconu et al. [24] adopted this formulation when the snap-through was triggered by a vertical load, concluding that the simple model of Dano and Hyer [22] over predicted the snap-through loads compared with the Finite Element Model (FEM). Schultz and Hyer [25,26] developed a model of the laminate-piezoceramic actuator combination to predict the shapes and the voltage needed to cause the snap-through and back. To be able to capture the snap-back of a cross-ply laminate, they proposed a refined third-order polynomial to model out-of-plane displacements.

Potter et al. [27] experimentally analysed the bifurcation of unsymmetric plates and they observed that the snap-through phenomenon was not a simple event and it occurred as a result of changes in curvatures. High order polynomials captured some intermediate snap-through points of this multi-event [28] using numerical routines based on path-following algorithms. The number of parameters, difficulties of resolution and the ratio between computational time and accuracy increased [28] when the order of polynomial increased.

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To measure the peak load in snap-through tests, the displacements at peak load and the energy necessary for the snap-through, some experimental procedures have been proposed. In the experimental test carried out by Potter et al. [27], the load was vertically introduced via a steel ball and the laminate was placed on a polished aluminium plate. This measurement procedure of the load–displacement curves has been used to compare predictions of FEM in dry and moist states [29] and to study the environmental effects on changes of curvatures and snap-through loads [7]. Tawfik et al. [30,31] carried out controlled experiments using vertical forces to trigger snap-through in the tested panels. Initially, the load was applied by a wide plunger [30]. Afterwards, an experimental setup [31] was established incorporating a load application rod and minimising the effect of friction through the use of air cushion. Zhang et al. [32,33] developed a two points loading method to capture the bi-stable behaviour of anti-symmetric composite shells based on [31].

In the previous references, cross-ply laminates analysed were supported on two edges during the snap-through process. To the best knowledge of the authors, the snap process of a cross-ply laminate trigged by a vertical force and supported at four points has not been previously analysed using the Rayleigh–Ritz technique. Therefore, the aim of this work is to propose a model to study snap-through and snap-back in square cross-ply laminates supported at four points and acted on by a concentrated load, based on the Rayleigh–Ritz technique and the minimisation of the TPE. To evaluate the strain energy of the whole laminate, four displacement fields are proposed: one corresponding to points among supports, and the others for points outside the supports. The model also considers a uniform through-the-thickness strain and non-uniform curvatures based on a previous work of the authors of the present study [34]. To validate the present approach, experimental tests have been carried out in cross-ply laminates supported on 4 rods and acted on by a central force, as it is shown in Fig. 2.

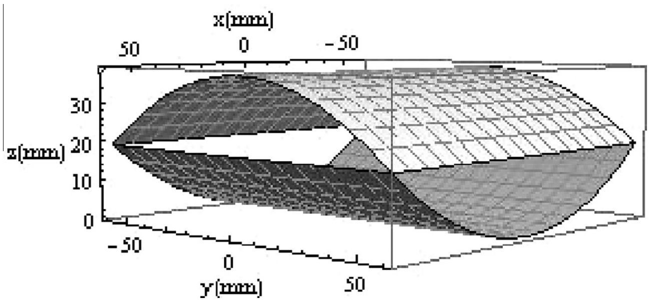

Fig. 1. Two stable configurations of a cross-ply laminate.

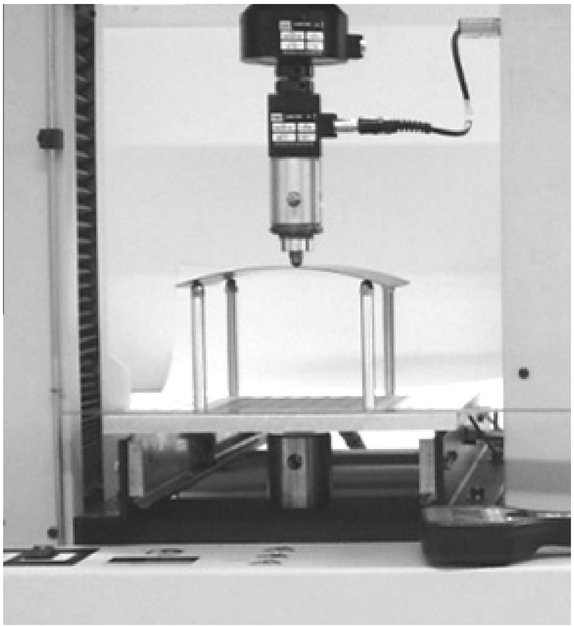

Fig. 2. Test configuration.

2. Proposed model

2.1. Motivation of the present approach

In the first approach to the problem of determination of the displacement field due to residual stresses in cross-ply laminates, Hyer [1,2,17] used the polynomial form of the vertical displacement field obtained by the Classical Lamination Theory (CLT), considering that the coefficients were unknown. Geometrically nonlinear terms of von-Kármán [17] were included in the mid-plane strains according to:

$$\begin{aligned}
\varepsilon_x^0 &= \frac{\partial u^0}{\partial x} + \frac{1}{2}\left(\frac{\partial w^0}{\partial x}\right)^2 \\
\varepsilon_y^0 &= \frac{\partial v^0}{\partial y} + \frac{1}{2}\left(\frac{\partial w^0}{\partial y}\right)^2 \qquad (1) \\
\gamma_{xy}^0 &= \frac{\partial u^0}{\partial y} + \frac{\partial v^0}{\partial x} + \frac{\partial w^0}{\partial x}\frac{\partial w^0}{\partial y}
\end{aligned}$$

The unknown coefficients of the polynomials were determined by minimising the TPE of the laminate.

Following the same idea of Hyer, in the present study the polynomial forms of the vertical displacement field has been obtained based on a previous approach concerning a plate supported at four points and acted on by a central load, valid for small displacements.

2.2. Displacements and strain fields

Mujika [35] developed an analytical approach of multidirectional laminates when a vertical force was applied at the centre of the laminate and the laminate was supported at four points. The laminate was divided into four parts, *A*, *B*, *C*, *D* as shown in Fig. 3. Due to central symmetry [36] the displacement field of *A*, *D* parts and *B*, *C* parts were the same. The out-of-plane displacement terms of a multidirectional laminate at any point of parts *A*,

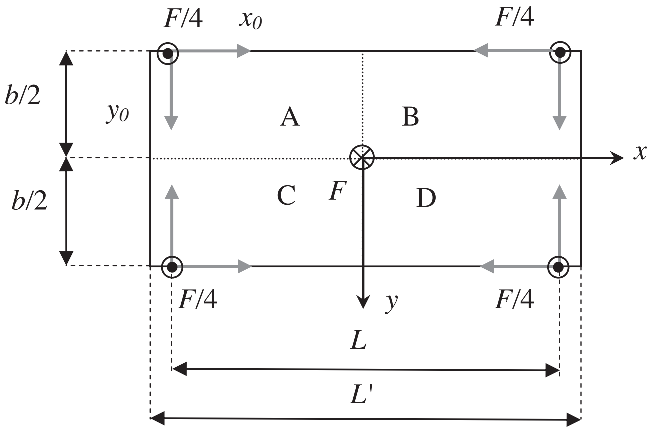

Fig. 3. Upper view of the plate used in the analytical approach of Mujika.