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# Semi-analytical buckling analysis of omega stiffened panels under multi-axial loads

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ABSTRACT

This paper presents a semi-analytical method for the buckling analysis of composite panels reinforced with omega stiffeners and subjected to combined loading conditions of biaxial loads and shear. The approach is based on the representation of the panel as an assembly of plate elements, allowing to capture local buckling modes involving skin and stiffener deflections. The panel model includes also the possibility of accounting for the stiffener foot. Trigonometric shape functions are introduced to describe the buckling patterns, while the buckling equations are derived through the application of the minimum potential energy principle. The comparison with Abaqus finite element analyses is presented, demonstrating, for a wide variety of test cases, percent differences below 9% and a good accuracy of the computed buckling modes. The computational speed up is of the order of 100, suggesting the use of the formulation in the context of preliminary design loops, sensitivity analyses or design optimizations.



## 1. Introduction

Composite stiffened panels are commonly used in load-bearing components of aerospace structures. As they usually operate under combined loads that can promote buckling phenomena, the ability to accurately predict the local buckling is an essential task since the preliminary design phases.

The most common approach to evaluate the buckling load is based on the finite element method [1,2], whose advantages rely on the accuracy of the results, as well as the possibility of accounting for various boundary and loading conditions. However, a major drawback is represented by the time to perform the analyses, which can be inappropriate in the early design stages. Furthermore, geometric modeling is required, and finite element meshes have to be created for each configuration under investigation.

The finite strip method is a numerical procedure specifically developed for the analysis of prismatic structures [3] and is characterized by an improved computational efficiency. The method has been successfully applied to assess the buckling behaviour of stiffened panels [4], to study the effects of different stiffener geometries [5], and to perform structural optimizations with buckling requirements [6].

An even more efficient approach consists in the use of ad hoc developed analytical formulations. In this case, the buckling load is calculated in a reduced time, so that sensitivity studies and design optimizations can be easily performed [7–9]. The accuracy of the results can be strongly affected by several assumptions regarding the modeling of the panel, the loading conditions and the stacking sequences. Some formulations available in literature simplify the stiffened panels as isolated plates with simply-supported or clamped constraints along the skin edges. The introduction of this assumption allows to derive, in the case of compression loads, the expression for the buckling load in a closed-form manner. The solutions are available for panels of isotropic and orthotropic [10] materials, as well as symmetric [11] and unsymmetric [12] laminates. More complex loading conditions are usually handled with semi-analytical strategies. For instance, combined loads are studied by Chai and Hoon [13] for simply-supported panels, while linearly varying biaxial in-plane loads are accounted for in the formulation of Romeo and Frulla [14]. Similarly, Shufrin et al. [15] consider general in-plane conditions by applying the extended method of Kantorovich. The assumption of simply-supported or clamped edges does not account for the finite amount of restraint provided by the stiffeners and, for this reason, can be the source of inaccurate results. In some other studies, the stiffener is modeled as a torsion spring or a De Saint–Venant torsion bar. Even in this case, the closed-form solutions are generally limited to the case of compression loads. Closed-form solutions are derived for orthotropic panels also by Bisagni and

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Vescovini [16] and Mittelstedt and Beerhorst [17], representing the stiffeners as torsion bars and torsion springs, respectively. Biaxial compression, in-plane bending and shear loads are introduced in the semi-analytical formulation of Bedair [18] for isotropic panels partially restrained against rotation and in-plane translation. Wittenberg et al. [19] investigate the shear buckling of stiffened panels with orthotropic skin lay-up, considering the flexural and torsional stiffness of the stiffeners with a formulation based on the method of Galerkin. In general, the methods accounting for the restraint to the edge rotation guarantee more accurate buckling predictions in comparison to those based on the assumption of simply-supported and clamped conditions. However, they still furnish a simplified description of the interaction between the skin and the stiffener, and local stiffener buckling modes cannot be captured.

The methods where both the skin and the stiffeners are modeled as two-dimensional plate elements allow for a more accurate representation of the skin-stiffener interaction, and both skin and local stiffener instabilities can be accounted for. The formulation of Fujikubo and Yao [20] derives closed-form solutions for the buckling load of isotropic stiffened panels, using a plate assembly model and energy principles and considering compression loads only.

Semi-analytical strategies are developed by Byklum and Amdahl [21] and Buermann et al. [22] in the context of the large deflection analysis of J-stiffened panels. They consider multiaxial loading conditions of compression and shear loads for isotropic materials. Composite materials were considered by the authors [23] in a formulation where double sines functions were used to represent the out of plane displacements. In general, all the mentioned methods introduce the simplifying assumption of neglecting the foot of the stiffener, i.e. the flange connecting the stiffener to the skin.

The present work discusses a plate assembly model for the buckling analysis of composite omega stiffened panels. Compared to most of the other methods in literature, the formulation introduces the possibility of considering the presence of the foot of the stiffener. The spectrum of loading conditions and laminate lay-ups is extended to include biaxial tension/compression and shear. Flexural anisotropy is accounted for, so that a wide class of typical aeronautical laminates can be studied.

The formulation is based on the minimum potential energy principle, which is applied referring to the method of Ritz. The governing equations are derived analytically and the computation of the buckling load is reduced to the solution of a set of eigenproblems of small dimension. The design tool is applied to study a series of panels, presenting the comparison with the finite element results in terms of buckling loads, buckled configurations and interaction curves. The development of the formulation is discussed throughout the paper, including considerations on the symmetry and the anti-symmetry of the buckling shapes, making possible a significant reduction of the computational effort.

## 2. Formulation

The work deals with the study of the buckling behaviour of omega flat stiffened panels made of composite materials and subjected to combined in-plane loads. The formulation considers the case of a large structure undergoing local buckling so that the analysis can be performed referring to a smaller unit, representative of the behaviour of the overall structure [21–23]. In particular, the unit is composed of two stiffeners, one bay and two half-bays, as reported in Fig. 1(a). The transverse edges of the panel are simply-supported, as the effect of the elasticity along these sides has a minor impact on the panel buckling load, while the longitudinal edges are subjected to periodic boundary conditions.

Only local buckling modes are investigated. The possible local buckling models present nodal lines along the intersections of the plate elements composing the panel. Global buckling modes, i.e. modes characterized by halfwaves encompassing several stiffeners, are not investigated.

The main idea of the present formulation is to represent the panel with a small number of plate elements, whose out of plane deflections are described with trigonometric shape functions. By introducing considerations regarding the symmetry and the anti-symmetry of the buckling modes, the model of Fig. 1(a) is reduced to the four plate element model of Fig. 1(b). The IDs of the four elements are highlighted in the figure together with the longitudinal length $a$, which is common to all the elements composing the panel.

The generic plate element $i$ is reported in Fig. 2. A Cartesian coordinate system is taken over the midsurface of each plate element. The $x$-and $y$-axis are directed along the longitudinal and the transverse directions, while the midsurface corresponds to $z = 0$. The figure illustrates also the loading conditions of biaxial compression (or tension) and shear, that can be applied separately or in combination.

The panels here analyzed are thin, and the transverse shear strains are assumed negligible. They are composed of laminates with an arbitrary number of layers, under the assumptions of symmetric lay-up, i.e. $B_{ik} = 0$, and null membrane anisotropy, i.e. $A_{16} = A_{26} = 0$. On the other hand, the bending-twisting coupling terms $D_{16}$ and $D_{26}$ are not neglected, so that the formulation can be applied to the study of panels made of symmetric and balanced laminates. The constitutive equation of the generic plate is then [10]:

$$\begin{Bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \\ M_{xx} \\ M_{yy} \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 & 0 & 0 & 0 \\ A_{12} & A_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & A_{66} & 0 & 0 & 0 \\ 0 & 0 & 0 & D_{11} & D_{12} & D_{16} \\ 0 & 0 & 0 & D_{12} & D_{22} & D_{26} \\ 0 & 0 & 0 & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ 2\epsilon_{xy} \\ -w_{,xx} \\ -w_{,yy} \\ -2w_{,xy} \end{Bmatrix} \tag{1}$$

where the terms $A_{ik}$ and $D_{ik}$ are the in-plane and bending stiffnesses, $N_{ik}$ and $M_{ik}$ are the internal forces and the moments per unit length, $\epsilon_{ik}$ are the components of the membrane strain tensor, and $w$ is the out of plane displacement. The comma followed by the coordinate denotes the differentiation with respect to that coordinate.

### 2.1. Total potential energy

The problem is formulated referring to the principle of the minimum potential energy, and the method of Ritz is applied to obtain an approximate solution. The main advantage of adopting a variational formulation relies in the need of satisfying only the essential boundary conditions of the problem. Indeed, the natural conditions involving the equilibrium of the forces exchanged between the adjacent plates elements could be hardly fulfilled, as it would be required in the context of a strong form formulation.

The total potential energy of the panel is given by the sum of the contributions of the plate elements composing the section, so:

$$\Pi = \sum_{i}^{\mathrm{N_p}} \Pi_i + \sum_{i}^{\mathrm{N_c}} P_i \tag{2}$$

where $\mathrm{N_p}$ is the number of plates to discretize the section, $\mathrm{N_c}$ is the number of compatibility conditions, and $P_i$ is a penalty term which is added to the functional to enforce the compatibility conditions between the adjacent plates. The first term in the right-hand side of Eq. (3) is: