Composite Structures 120 (2015) 300–314

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

Coupled stretching–bending analysis of laminated plates with corners via boundary elements

Chyanbin Hwu *, H.W. Chang

Institute of Aeronautics and Astronautics, National Cheng Kung University, Tainan, Taiwan, ROC

ARTICLE INFO

Article history:
Available online 13 October 2014

Keywords:
Boundary element
Coupled stretching–bending analysis
Laminated plate
Corner
Singular integral
Auxiliary equation

ABSTRACT

To cover all the possibility of symmetric, anti-symmetric, or unsymmetric thin laminated plates, the fundamental solutions used in the present boundary element was obtained from the Green's functions for the coupled stretching–bending analysis. Considering the possible existence of corners in thin laminated plates, four different approaches were proposed and compared to deal with the corners: (1) round-off corners, (2) triple independent nodes on the boundary, (3) triple independent nodes outside the boundary, and (4) triple nodes with the same position on the corner. Due to the equation dependency raised by the same position of the fourth approach, four auxiliary equations suited for various boundary conditions were derived by considering the symmetry of stress tensor and using the laminate constitutive relations. The numerical results show that the last two approaches perform well in all examples, whereas the first two fail in certain cases of the unsymmetric laminates.

1. Introduction

In the classical thin plate theory, in addition to the edge forces there may be concentrated forces produced at the corners [1]. Thus, the additional terms related to the corner forces should be included in the boundary element formulation for plate bending analysis [2]. Same situation occurs for the more complicated coupled stretching–bending analysis for the general laminated plates [3,4]. With the addition of the corner forces, the problems involved in the corners of thin laminated plates become much more complicated. To avoid the troubles induced by the corners, at the beginning of boundary element analysis the rounding-off procedure was usually employed to get reasonable answers away from the rounded region [5]. However, this procedure cannot be considered as a satisfactory solution to all the problems with corners since the results near or at some distance away from the corners must be affected. Detailed and precise analyses are always required to provide more accurate results.

For two or three dimensional problems, several methods have been proposed in the literature to deal with the problems of corners such as moving the source points to the outside of body [6–10], multiple independent node concept [11–13], multiple-node concept with auxiliary relationships [13–17], and special kernel functions for the corners [18]. As far as the authors' knowledge, none of them has been employed for the coupled stretching–bending analysis via boundary element method. Especially, the auxiliary relationships for the corners published in the literature are only valid for the problems with isotropic materials, which will not encounter the complexity of the coupling between in-plane problem and plate bending problem. However, for the use of unsymmetric composite laminates, the coupled stretching–bending analysis cannot be avoided. In this paper, to know whether these methods can be extended to the coupled stretching–bending analysis, four different approaches are employed to deal with the corners, i.e., (1) rounding-off corners, (2) triple independent nodes on the boundary, (3) triple independent nodes outside the boundary, and (4) triple nodes with the same position on the corner. Since in the fourth approach the equations produced by the triple nodes of corners may be dependent of each other due to the same position, additional auxiliary equations are necessary for complete implementation. By considering the symmetry of stress tensor and the connection between nodal displacements and nodal forces through laminated constitutive relations, in this paper four auxiliary equations were derived to supplement the lack of equations for the corner nodes. When reducing to the cases of two-dimensional isotropic elastic solids, two of these four auxiliary equations have been proved to be the same as those obtained by Chaudonneret [13].

To see the performance of these four different approaches, three numerical examples covering several different kinds of laminates with various boundary conditions subjected to different types of

* Corresponding author. Tel.: +886 6 2757575; fax: +886 6 2389940.
E-mail address: CHwu@mail.ncku.edu.tw (C. Hwu).

http://dx.doi.org/10.1016/j.compstruct.2014.09.066
0263-8223/

loadings are illustrated. The validity and accuracy of these approaches were verified by comparing with the analytical solutions of some benchmark problems such as pure tension and pure bending [19], and the numerical solutions calculated by the other methods [20], or commercial finite element software ANSYS. Through the comparison of the numerical results, the fourth approach using the multiple-node concept with auxiliary relationships always performs well in all situations. Moreover, using this approach, we do not need to worry about the location of the triple nodes because they always locate on the same point of the corner. For the other three approaches, the adjustment and convergence study are always needed for the arrangement of the three corner nodes.

2. Coupled stretching–bending analysis of general laminates

To describe the overall properties and mechanical behavior of a laminate, the most popular way is the classical lamination theory [21]. According to the observation of actual mechanical behavior of laminates, Kirchhoff's assumptions are usually made in this theory. Based upon the Kirchhoff's assumptions, the displacement fields, the strain–displacement relations, the constitutive laws and the equilibrium equations, the governing equations controlling the mechanical behavior of the general laminates (see Fig. 1) can be written in terms of three unknown displacement functions u_0, v_0 and w as [3]

$$A_{11}\frac{\partial^2 u_0}{\partial x^2}+2A_{16}\frac{\partial^2 u_0}{\partial x\partial y}+A_{66}\frac{\partial^2 u_0}{\partial y^2}+A_{16}\frac{\partial^2 v_0}{\partial x^2}+(A_{12}+A_{66})\frac{\partial^2 v_0}{\partial x\partial y}+A_{26}\frac{\partial^2 v_0}{\partial y^2}-B_{11}\frac{\partial^3 w}{\partial x^3}-3B_{16}\frac{\partial^3 w}{\partial x^2\partial y}-(B_{12}+2B_{66})\frac{\partial^3 w}{\partial x\partial^2 y}-B_{26}\frac{\partial^3 w}{\partial y^3}=0, \quad (2.1a)$$

$$A_{16}\frac{\partial^2 u_0}{\partial x^2}+(A_{12}+A_{66})\frac{\partial^2 u_0}{\partial x\partial y}+A_{26}\frac{\partial^2 u_0}{\partial y^2}+A_{66}\frac{\partial^2 v_0}{\partial x^2}+2A_{26}\frac{\partial^2 v_0}{\partial x\partial y}+A_{22}\frac{\partial^2 v_0}{\partial y^2}-B_{16}\frac{\partial^3 w}{\partial x^3}-(B_{12}+2B_{66})\frac{\partial^3 w}{\partial x^2\partial y}-3B_{26}\frac{\partial^3 w}{\partial x\partial y^2}-B_{22}\frac{\partial^3 w}{\partial y^3}=0, \quad (2.1b)$$

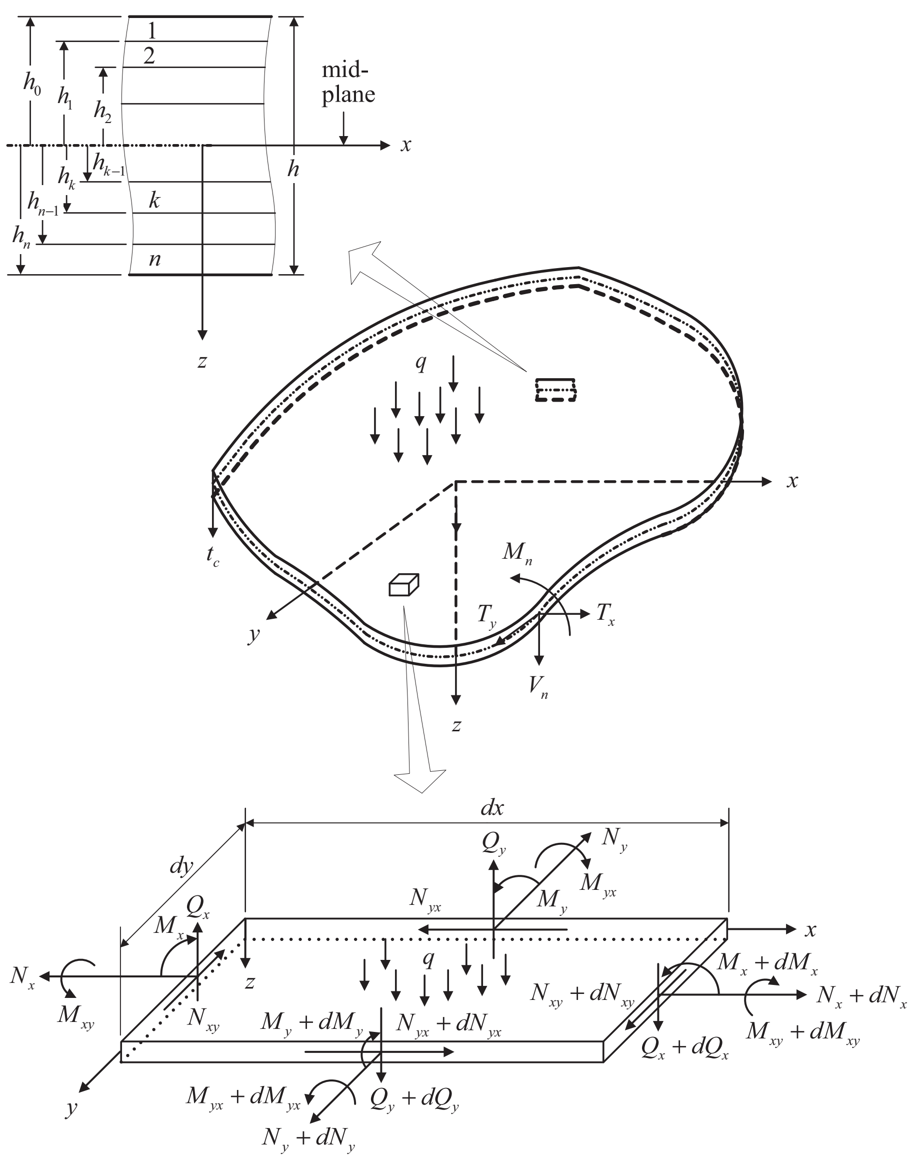

Fig. 1. Laminate geometry, stress resultants and bending moments.