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A sampling surfaces method and its implementation for 3D thermal stress analysis of functionally graded plates

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ABSTRACT

The paper deals with a recently developed method of sampling surfaces (SaS) and its implementation for the three-dimensional (3D) steady-state problem of thermoelasticity for laminated functionally graded (FG) plates subjected to thermomechanical loading. The SaS method is based on choosing inside the nth layer I_n not equally spaced SaS parallel to the middle surface of the plate in order to introduce temperatures and displacements of these surfaces as basic plate variables. Such an idea permits the presentation of the thermoelastic laminated FG plate formulation in a very compact form. The SaS are located inside each layer at Chebyshev polynomial nodes that provides a uniform convergence of the SaS method. This means that the SaS method can be applied efficiently to the 3D stress analysis for thermoelastic laminated FG plates with a specified accuracy utilizing the sufficient number of SaS.

1. Introduction

Nowadays, the functionally graded (FG) materials are widely used in mechanical engineering due to their advantages compared to traditional laminated materials [1,2]. However, the study of FG materials is not a simple task because the material properties depend on the spatial coordinate and some specific assumptions regarding their continuous variations in the thickness direction are required [3]. This fact restricts the implementation of the Pagano approach [4,5] and the state space approach [6,7] for the 3D exact analysis of FG simply supported rectangular plates. Another popular approach to 3D exact solutions, namely, asymptotic approach was applied efficiently to FG plates subjected to thermomechanical loading [8,9]. A new approach to closed-form elasticity solutions for FG isotropic and transversely isotropic plates is considered in papers [10,11]. These solutions are based on the general solution of the equilibrium equations of inhomogeneous elastic media [12]. The efficient approach to the 3D exact analysis of thermoelasticity has been proposed by Vel and Batra [13,14]. They studied the static and transient thermoelastic problems for FG simply supported plates with the material properties presented by Taylor series expansions through the thickness coordinate. Ootao and his coauthors [15–17] obtained the 3D exact solutions for the transient thermoelastic response of FG strips and rectangular plates with simply supported edges under nonuniform heating on outer surfaces. The original approach to analytical solutions for the FG beams and plates was developed in contributions [18,19]. This approach is based on the so-called theory of directed surfaces [20,21]. Recently, the sampling surfaces (SaS) approach has been applied to 3D exact thermal and thermoelastic analyses of laminated composite plates and shells [22–24]. The 3D stress analysis of piezoelectric FG plates and shells on the basis of the SaS method is given in [25,26]. However, the SaS approach has not been applied to 3D steady-state thermoelasticity problems for laminated FG plates yet.

According to the SaS method [27,28], we choose inside the nth layer I_n not equally spaced SaS $\Omega^{(n)1}, \Omega^{(n)2}, \ldots, \Omega^{(n)I_n}$ parallel to the middle surface of the plate and introduce temperatures $T^{(n)1}, T^{(n)2}, \ldots, T^{(n)I_n}$ and displacement vectors $\mathbf{u}^{(n)1}, \mathbf{u}^{(n)2}, \ldots, \mathbf{u}^{(n)I_n}$ of these surfaces as basic plate variables, where $I_n \geqslant 3$. Such choice of unknowns in conjunction with the use of the Lagrange polynomials of degree $I_n - 1$ in the thickness direction permits the presentation of governing equations of the proposed thermoelastic FG plate formulation in a very compact form.

It should be mentioned that the SaS method with equally spaced SaS does not work properly with the Lagrange polynomials of high degree because of the Runge's phenomenon [29]. This phenomenon can yield the wild oscillation at the edges of the interval when the user deals with any specific functions. If the number of equally spaced nodes is increased then the oscillations become even larger. However, the use of the Chebyshev polynomial nodes [30] inside each layer can help to improve significantly the behavior of the Lagrange polynomials of high degree because such a

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choice allows one to minimize uniformly the error due to the Lagrange interpolation.

Currently, the use of layer-wise theories for the analysis of laminated composite plates is widely accepted. The most general form of layer-wise kinematics presented in Carrera's unified formulation [31] is written as

$$u_i^{(n)} = F_0 u_i^{[n-1]} + F_1 u_i^{[n]} + \sum_r F_r u_{ir}^{(n)}, \quad x_3^{[n-1]} \leqslant x_3 \leqslant x_3^{[n]},$$

$$F_0(x_3) = \frac{x_3^{[n]} - x_3}{h_n}, \quad F_1(x_3) = \frac{x_3 - x_3^{[n-1]}}{h_n}, \quad F_r\left(x_3^{[n-1]}\right) = F_r\left(x_3^{[n]}\right) = 0,$$

where $u_i^{(n)}(x_1, x_2, x_3)$ are the displacements of the nth layer ($i = 1,2,3$); $u_i^{[n-1]}(x_1, x_2)$ and $u_i^{[n]}(x_1, x_2)$ are the displacements of the bottom and top surfaces of the nth layer (interfaces); $u_{ir}^{(n)}(x_1, x_2)$ are the generalized displacements of the nth layer ($r = 2,3,\ldots,R$); $F_r(x_3)$ are the prescribed polynomials of degree r; $x_3^{[n-1]}$ and $x_3^{[n]}$ are the transverse coordinates of layer interfaces $\Omega^{[n-1]}$ and $\Omega^{[n]}$ (Fig. 1); $h_n = x_3^{[n]} - x_3^{[n-1]}$ is the thickness of the nth layer; x_1 and x_2 are the Cartesian coordinates of the middle surface Ω; x_3 is the thickness coordinate normal to the middle surface; the index n identifies the belonging of any quantity to the nth layer and runs from 1 to N, where N is the number of layers. Historically, the first order layer-wise models [32–35] were first. Then, the second order models with $R = 2$ and third order models with $R = 3$ were developed [36–38]. The fourth order layer-wise model ($R = 4$) is utilized in Carrera's unified formulation [39–41], where polynomials F_r are evaluated as a difference between two Legendre polynomials of degrees r and $r - 2$.

The origins of using the SaS can be found in contributions [42,43] in which three, four and five equally spaced SaS are employed. The SaS method with the arbitrary number of equispaced SaS is considered in [44]. The more general approach with the SaS located at Chebyshev polynomial nodes has been developed later [27,28]. Note also that the term SaS should not be confused with such terms as a mathematical surface or a virtual surface, which are extensively utilized in Carrera's unified formulation. This is due to the fact that in Carrera's unified formulation the generalized displacements of layers $u_{ir}^{(n)}$ are employed. On the contrary, in a developed SaS formulation all basic variables have a clear mechanical sense because of the introduction of temperatures and displacements of SaS as plate unknowns. The similar technique is adopted for the description of material properties, which are also referred to SaS. This gives the opportunity to derive the 3D exact solutions for laminated FG plates with a prescribed accuracy utilizing the sufficiently large number of SaS located at Chebyshev polynomial nodes inside each layer. Furthermore, in a SaS formulation for shells such choice of displacements as fundamental unknowns yields the strain–displacement equations, which exactly represent rigid-body motions of the shell in any convected curvilinear coordinate system [28]. The latter is straightforward for development of the exact geometry solid-shell elements [45,46]. The term "exact geometry" reflects the fact that the parametrization of the middle surface is known and, therefore, the coefficients of the first and second fundamental forms of its surface can be taken exactly at each element node.

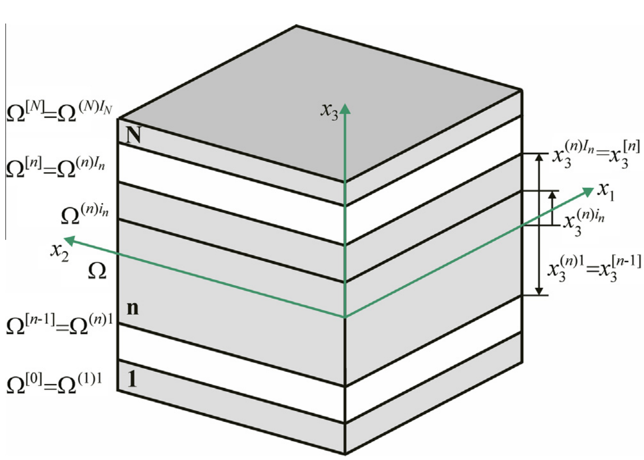

Fig. 1. Geometry of the laminated plate.

2. Description of temperature and temperature gradient fields

Consider a laminated FG plate of the thickness h. The transverse coordinates of SaS of the nth layer are defined as

$$x_3^{(n)1} = x_3^{[n-1]}, \quad x_3^{(n)I_n} = x_3^{[n]},$$
$$x_3^{(n)m_n} = \frac{1}{2}\left(x_3^{[n-1]} + x_3^{[n]}\right) - \frac{1}{2} h_n \cos\left(\pi \frac{2m_n - 3}{2(I_n - 2)}\right), \tag{1}$$

where I_n is the number of SaS corresponding to the nth layer; the index m_n identifies the belonging of any quantity to the inner SaS of the nth layer and runs from 2 to $I_n - 1$, whereas the indices i_n, j_n, k_n to be introduced later for describing all SaS of the nth layer run from 1 to I_n. Besides, the tensorial indices i,j,k,l range from 1 to 3 and Greek indices α, β range from 1 to 2.

Remark 1. The transverse coordinates of inner SaS (1) coincide with coordinates of the Chebyshev polynomial nodes [30]. This fact has a great meaning for a convergence of the SaS method [22–28].

The relation between the temperature T and the temperature gradient $\boldsymbol{\Gamma}$ is given by

$$\boldsymbol{\Gamma} = \nabla T. \tag{2}$$

In a component form, it can be written as

$$\Gamma_i = T_{,i}, \tag{3}$$

where the symbol $(\ldots)_{,i}$ stands for the partial derivatives with respect to coordinates x_i.

We start now with the first and second fundamental assumptions of the proposed thermoelastic laminated plate formulation. Let us assume that the temperature and temperature gradient fields are distributed through the thickness of the nth layer as follows:

$$T^{(n)} = \sum_{i_n} L^{(n)i_n} T^{(n)i_n}, \quad x_3^{[n-1]} \leqslant x_3 \leqslant x_3^{[n]}, \tag{4}$$

$$\Gamma_i^{(n)} = \sum_{i_n} L^{(n)i_n} \Gamma_i^{(n)i_n}, \quad x_3^{[n-1]} \leqslant x_3 \leqslant x_3^{[n]}, \tag{5}$$

where $T^{(n)i_n}(x_1, x_2)$ are the temperatures of SaS of the nth layer $\Omega^{(n)i_n}$; $\Gamma_i^{(n)i_n}(x_1, x_2)$ are the components of the temperature gradient at the same SaS; $L^{(n)i_n}(x_3)$ are the Lagrange polynomials of degree $I_n - 1$ defined as

$$T^{(n)i_n} = T\left(x_3^{(n)i_n}\right), \tag{6}$$

$$\Gamma_i^{(n)i_n} = \Gamma_i\left(x_3^{(n)i_n}\right), \tag{7}$$

$$L^{(n)i_n} = \prod_{j_n \neq i_n} \frac{x_3 - x_3^{(n)j_n}}{x_3^{(n)i_n} - x_3^{(n)j_n}}. \tag{8}$$

The use of Eqs. (3), (4), (6) and (7) yields

$$\Gamma_\alpha^{(n)i_n} = T_{,\alpha}^{(n)i_n}, \tag{9}$$

$$\Gamma_3^{(n)i_n} = \sum_{j_n} M^{(n)j_n}\left(x_3^{(n)i_n}\right) T^{(n)j_n}, \tag{10}$$