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Analytical homogenization for stretch and bending of honeycomb sandwich plates with skin and height effects

Y.M. Li *, M.P. Hoang, B. Abbes, F. Abbes, Y.Q. Guo

GRESPI/MPSE, Université de Reims Champagne-Ardenne, Campus du Moulin de la Housse, BP 1039, 51687 Reims Cedex 2, France

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ABSTRACT

The honeycomb sandwich plates are widely used in the automotive, aeronautic and aerospace industries. However, the numerical modeling of honeycomb structures is too tedious and time consuming. The homogenization of these structures enables to obtain an equivalent homogeneous solid and its elastic moduli thus to make very efficient simulations. In the present study, the skin effect is taken into consideration for the stretching and bending problems, in which the two skins are much more rigid than the honeycomb core. An analytic homogenization method, using trigonometric function series is proposed to study the influence of the honeycomb height on the elastic properties, and the upper and lower bounds of the equivalent elastic moduli of their curves are found compatible with those obtained by the membrane theory. The interfacial stresses are also studied in order to compare with the existing model for the stretching problem. Numerical H-models are established for the stretching, bending and the stretch–bending coupled problem respectively, and a very good agreement has been achieved between the results of the present H-models and 3D FE modeling.

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1. Introduction

The honeycomb sandwich plates are widely used in the automotive, aeronautic and aerospace industries. The simulation and optimization of this kind of plates are of prime importance for the lightness and safety of structures. However, the numerical modeling of honeycomb structures is too tedious and time consuming due to the complexity of the structure. The homogenization of these structures enables to obtain an equivalent homogeneous solid and make the simulations much more efficient. Some experimental and numerical work has been carried out to determine the global elastic properties [1,2]. As the geometry and boundary conditions of honeycomb structures are complex, the analytical homogenization is an efficient method.

Many studies have been performed on the analytical homogenization of honeycomb structures [3,4]. The book of Gibson and Ashby [5] is the first systematic literature in this field. The in-plane elastic properties of honeycomb were first obtained with the beam theory. Further refinements by Masters and Evans [6] have been attempted considering stretching and hinging effects. However, all these mathematical models on honeycomb cores are based on pure cellular structures without considering the strengthening effect of the skin faces. In the classical sandwich theory [7], the global skin-core interaction is identified as the result of the anti-plane core assumption. Since the two skins' constraints alter significantly the local deformation mechanism of the core, the homogenized core properties become sensitive to the core thickness over the unit cell size ratio, which is called thickness effect by Becker [8,9]. Becker has studied the displacement behaviors of the honeycomb core in two limiting cases, namely directly at the interface of the core and skins, and on the mid surface of the core far away from the skins; then he proposed an interpolation function between these two cases in order to describe the displacement field through the core height between the skins.

Another interesting approach has been proposed by Xu et al. [10,11] to homogenize a honeycomb unit cell including the skin effect. Firstly, the homogenization was carried out along *X*-direction (Fig. 1) to obtain two homogenous solids corresponding to inclined and vertical walls, called Part I and Part II; then a second homogenization was carried out along *Y*-direction to obtain the whole homogenous solid. Based on the asymptotic expansions and characteristic periodicity, the displacement functions between the two solids were formulated and analytically resolved. However, the interactions between the two homogenized solids were not equivalent to those between the vertical and inclined walls; furthermore the Poisson coefficients in Part I and Part II were not treated together

* Corresponding author.
E-mail address: yuming.li@univ-reims.fr (Y.M. Li).

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with the stretch moduli, leading to some errors. Lately Chen et al. [12] have established another analytical model to calculate the moduli and the interfacial stresses for stretching problems. A solution using trigonometric function series based on the equilibrium equations has been proposed under the assumptions of some boundary conditions. Both Xu [10,11] and Chen [12] have used the function *sinh* in the displacement field that causes numerical problem when the variable of the function *sinh* is large whereas its variable reaches ∞ theoretically. Due to the fact that the honeycomb bending deformation cannot be treated as a plate bending problem using the equivalent in-plane elastic modulus, Chen [13] has proposed a method for an open honeycomb to calculate the honeycomb flexural stiffness based on the bending and twisting deformation of each plate forming the honeycomb. The theory of thin plate was used for the theoretical solution of bending deflection w to the problem, but the skin effects were not studied whereas they are important.

In the present study, we limit ourselves to the stretching and bending problems of honeycomb sandwich plates in which the two skins are much more rigid than the honeycomb core walls. Thus, for the in-plane stretching problem it is assumed that: (1) the core has little influence on the skin's behavior, and its deformation is constrained by the two skins; (2) the stretching rigidities of a honeycomb core are essentially given by its thin walls' stretch instead of its walls' bending. The energetic homogenization method is used to determine the elastic properties of the homogenized solid. In a representative element volume on a unit cell (REV), two analytical homogenization models based on the membrane theories are established to find the upper and lower bounds of the elastic moduli. To study the influence of the honeycomb core height on its elastic moduli, a homogenization formulation using some trigonometric function series is proposed by taking into account the stress redistribution between the honeycomb walls. The minimization of the internal strain energy of a REV enables to obtain the parameters of the trigonometric function series and then the elastic moduli. Furthermore, the above method may give the upper and lower bound of the moduli by using the height of honeycomb core tending to zero and to infinity, respectively. A method combining the stretch and bending moduli is proposed to deal with the stretch–bending coupled problems. The interfacial stresses are also studied and compared with Chen's model [12] for the stretching problem. For the bending problem, a similar homogenization formulation is established. Numerical models called "H-model" are established for stretch, bending and stretch–bending coupled problems to calculate the moduli of the homogenized solid based on the above method. And a very good agreement has been achieved between H-models and 3D FE modeling results.

2. Homogenization formulation for in-plane stretch moduli

A honeycomb cell (Fig. 1) is taken as Representative Elementary Volume (REV). In the classical homogenization theory [5], the stretching properties are determined only on a cell without the skin effect, and the properties depend only on the bending behavior of the thin walls of the honeycomb. In the present study, the skins are supposed very rigid, so the honeycomb walls are constrained by the skins, and the thin wall stretching effect is dominant with respect to its bending effect. Consequently for the regular hexagon honeycomb with identical thickness ($t' = 2t$ and $h = l$), the stretch moduli of the honeycomb cell are rather proportional to t/l (Fig. 1) instead of to $(t/l)^3$ that is showed in [5]. Taking $t = 0.19$ mm and $l = 8$ mm as an example, the stretch moduli with the skin effect are 591 times of those without skin effect! The deformation of the 1/8 honeycomb REV under a tensile load in Y-direction which is shown in Fig. 2 is due to the stress redistribution between the three walls.

2.1. Methodology to determine elastic moduli of a homogenized solid

The homogenization consists in replacing the honeycomb structure by an equivalent homogenized solid. The energetic homogenization method is used to determine the elastic properties of the solid.

The orthotropic constitutive law of the solid is expressed as follows (homogenized quantities are noted by $*$):

$$\begin{Bmatrix} \sigma_X^* \\ \sigma_Y^* \end{Bmatrix} = \begin{bmatrix} E_X^* & E_{XY}^* \\ E_{XY}^* & E_Y^* \end{bmatrix} \begin{Bmatrix} \varepsilon_X \\ \varepsilon_Y \end{Bmatrix} \tag{1}$$

In the above elastic matrix $[E^*]$, the Poisson coefficients are already coupled in the three moduli to be determined (E_X^*, E_Y^*, and E_{XY}^*). It is noted that in most of the honeycomb homogenization studies, the three stretch moduli were determined separately without respecting Eq. (1). That is, without considering their coupling effect.

The internal strain energy of the homogenized solid π_{int}^* can be obtained by using Eq. (1), and it should be equal to that of the honeycomb core π_{int}:

$$\pi_{int}^* = \frac{1}{2}\int_{\overline{V}} \langle \varepsilon \rangle [E^*] \{\varepsilon\} d\overline{V} = \pi_{int} \tag{2}$$

The above equation gives:

$$\varepsilon_X^2 E_X^* + 2\varepsilon_X \varepsilon_Y E_{XY}^* + \varepsilon_Y^2 E_Y^* = \frac{2\pi_{int}}{\overline{V}} \tag{3}$$

where $\overline{V} = Ab/2$ is the 1/8 volume of the REV, $A = l\cos\theta(h + l\sin\theta)$ is the 1/4 surface of the honeycomb REV, b is the height of the honeycomb, and ε_X and ε_Y define the strain field imposed by the skins.

In order to obtain the three moduli, in the case of the analytical models for the upper and lower bounds, the internal strain energy π_{int} in Eq. (3) can be also expressed in function of $\varepsilon_X^2, 2\varepsilon_X\varepsilon_Y$ and ε_Y^2, the comparison of the corresponding terms on both sides of Eq. (3) gives directly and explicitly the three unknown moduli E_X^*, E_Y^*, E_{XY}^*.

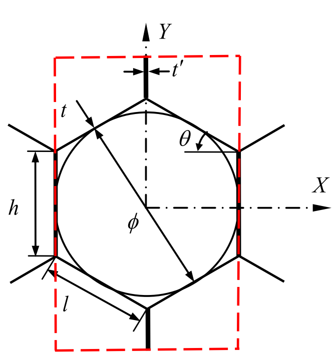

Fig. 1. Honeycomb REV.

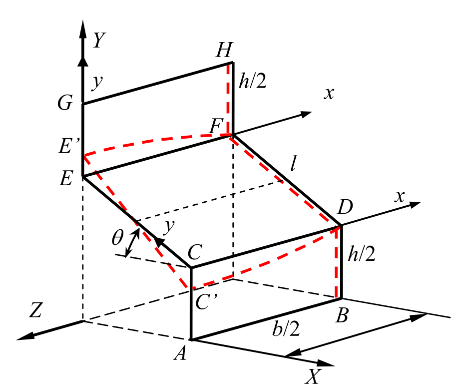

Fig. 2. Supplementary displacements due to stress redistribution.

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