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Nonlinear thermo-inertial instability of functionally graded shape memory alloy sandwich plates

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ABSTRACT

The nonlinear thermal instability of moving sandwich plates subjected to a constant moving speed is investigated. The sandwich plates are made of a shape memory alloy (SMA) fiber reinforced composite core and two functionally graded (FG) face sheets (FG/SMA/FG). The Brinson model is used for modeling of SMA behavior, and the geometrically nonlinear third-order shear deformation theory is used for modeling of sandwich plates. Thermomechanical properties of the sandwich plate are assumed to be temperature-dependent. Two types of in-plane boundary conditions are considered. The nonlinear instability is treated by the Galerkin technique. The results show that the initial equilibrium configuration of the sandwich plate becomes unstable at a critical moving speed. Results examine the effect of axial moving speed, SMA volume fraction, pre-strain of SMA fiber, core thickness, imperfection, in-plane boundary conditions, and thermal loading on the stability characteristics of the sandwich plates. It is found that a proper application of SMA fibers postpone the thermal buckling and critical moving speed. Furthermore, the induced tensile recovery stress of SMA fibers could also stabilize geometrically imperfect plates during the reverse martensite transformation.

1. Introduction

Axially moving plates can be used in a variety of industrial, mechanical, and aerospace applications, such as power transmission belts, robotic manipulators, and magnetic tapes [1]. The transverse vibration and lateral deflection of moving structures are major design concerns for moving components since a system could lose its stability by divergence (buckling) at a critical speed [1]. Due to the applications of high-speed moving structures in aeronautical and aerospace engineering, an investigation on the buckling stability of these structures is significant to prevent excessive drag caused by moving inertial force. Several studies have examined the dynamic and stability responses of moving structures [2–11]. For instance, the stability of thin isotropic plates moving with a high axial speed was presented by Luo and Hamidzadeh [9]. Asadi et al. [10] examined the dynamic response of a moving functionally graded (FG) plate with an internal rigid line support in various thermal conditions. The nonlinear dynamic response of moving plates was also studied by Ghayesh et al. [11] by using the pseudo-arclength continuation technique.

In addition extensive research studies on stability and dynamic analysis of stationary composite and FG structures have been conducted [12–20]. Akbarzadeh et al. [14,17] theoretically studied the transient thermoelastic responses of thick- and thin-walled FG cylinders and plates. The buckling behavior of a thick FG plate was studied by Bateni et al. [20] based on the refined plate theory.

Over the past decades, advances in the shape memory alloy (SMA) reinforced structures have drawn specific attention in the applications of SMAs in aerospace industry. Compared to the other intelligent materials, SMAs are more flexible in applications in intelligent structures because of their desired thermomechanical characteristics. These thermomechanical behaviors could be reflected by their extremely large recovery strains and the unique material properties that vary with load history, temperature, and stress. Moreover, SMAs are suitable for actuation applications due to their direct coupling of thermal and mechanical fields with high actuation frequency and actuation energy density [21]. There exists a considerable amount of research on the application of SMA layers/fibers in composite structures [22–34]. Among the primary research in this area, one may refer to the work of Rogers et al. [22], in which several concepts of structural improvement of

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SMA composite plates were formulated. Choi et al. [24] conducted a theoretical and experimental study on controlling the shape of SMA composite beams based on the Euler–Bernoulli beam theory. Tawfik et al. [34] later used a new finite element formulation to solve the thermal post-buckling of SMA quasi-isotropic plates. It was found that SMA fibers could substantially improve the post-buckling behavior of laminated composite plates.

While the abovementioned investigations covered the subject of linear and nonlinear dynamic and static responses of SMA reinforced composites. In all of these studies, thermomechanical responses of SMA fibers have not been extensively investigated, except few attempts that have extracted the approximate curves from experimental results. A few studies have been developed by using the phenomenological model to express the constitutive characteristics of SMA fibers [35–45]. Roh et al. [35] investigated the thermal snapping phenomenon in shell panels by the finite element method. Cho and Rhee [36] introduced a nonlinear finite element approach for modeling the hybrid laminated composite shell reinforced by SMA fibers subjected to the thermomechanical loading. Nonlinear dynamic and stability responses of laminated composite beams containing SMA fibers were investigated by Asadi et al. [39,40]. A study on the influence of SMA fibers on structural responses of hybrid composite plates was also conducted by Asadi et al. [41]. Results of this study reveal that the orientation of SMA fibers has a major effect on thermal buckling loads. In addition, a review on the structural enhancement by using SMA material was conducted by Wang and Wu [45].

To the best of the authors' knowledge, there is no work reported on the instability of axially moving FG/SMA/FG sandwich plates subjected to the thermo-inertial loading. The main objective of the present work is to investigate the nonlinear buckling stability of imperfect sandwich plates subjected to thermo-inertial loadings. The Brinson model is used to calculate the tensile recovery stress generated by SMA fibers. The geometrically nonlinear third-order shear deformation theory (TSDT) is employed to describe the displacement fields. The temperature dependency of thermomechanical properties is taken into account. Closed-form expressions are presented to obtain critical moving speed, critical buckling temperature, and the thermal bifurcation path. Parametric studies are also presented to provide an insight into the effects of moving speed, volume fraction and pre-strain of SMA fibers, geometrical imperfection, and temperature on the thermo-inertial instability of FG/SMA/FG sandwich plates.

2. FG/SMA/FG sandwich plates

A three-layer sandwich plate consisting of two FGM face sheets and an SMA fiber reinforced composite substrate is considered in Fig. 1. The thickness, width, and length of the sandwich plate are represented by h,b, and a, respectively. Following the power-law distribution in the z-direction, the volume fraction of the metal component of layers, V_m, is expressed as

$$V_m = \begin{cases} \left(\frac{2z+h}{2h_f}\right)^K & -\frac{h}{2} \leqslant z \leqslant -\frac{h_c}{2} \\ 1 - V_s & -\frac{h_c}{2} \leqslant z \leqslant \frac{h_c}{2} \\ \left(\frac{-2z+h}{2h_f}\right)^K & \frac{h_c}{2} \leqslant z \leqslant \frac{h}{2} \end{cases} \quad (1)$$

where V_s, h_f, and h_c stand for the volume fraction of SMA fibers, thickness of each FG sheet, and the thickness of composite substrate, respectively. The non-homogeneous material properties of FG face sheets are obtained by the rule of mixture as follows

$$\mathrm{P} = \begin{cases} \mathrm{P}_C + \mathrm{P}_{MC}\left(\frac{2z+h}{2h_f}\right)^K & -\frac{h}{2} \leqslant z \leqslant -\frac{h_c}{2} \\ \mathrm{P}_C + \mathrm{P}_{MC}\left(\frac{-2z+h}{2h_f}\right)^K & \frac{h_c}{2} \leqslant z \leqslant \frac{h}{2} \end{cases} \quad (2)$$

where, $\mathrm{P}_{MC} = \mathrm{P}_M - \mathrm{P}_C$, K is the power-law index, and P_C and P_M are thermomechanical properties of the ceramic and metal matrix, respectively. In order to accurately predict the thermomechanical behavior of FG sheets, the temperature dependency of material constituents is taken into account [16]

$$\mathrm{P}(T) = \mathrm{P}_0\left(\mathrm{P}_{-1}T^{-1} + 1 + \mathrm{P}_1 T + \mathrm{P}_2 T^2 + \mathrm{P}_3 T^3\right) \quad (3)$$

where P_i constants are given in Table 1.

Birman et al. [46] presented the extended multi-cell micromechanics approach [47] to calculate the equivalent properties of the composite systems consisting of SMA fibers embedded within an elastic matrix and in a uniform temperature field. Correspondingly, the material properties of the Aluminum/NiTi could be obtained as

$$\begin{aligned}
E_{11} &= E_s(\xi)V_s + E_m(1 - V_s) \\
E_{22} &= E_m\left[\left(1 - \sqrt{V_s}\right) + \frac{\sqrt{V_s}}{1 - \sqrt{V_s}\left(1 - \frac{E_m}{E_s(\xi)}\right)}\right] \\
G_{13} = G_{12} &= G_m\left[\left(1 - \sqrt{V_s}\right) + \frac{\sqrt{V_s}}{1 - \sqrt{V_s}\left(1 - \frac{G_m}{G_s(\xi)}\right)}\right] \\
G_{23} &= \frac{G_m}{1 - \sqrt{V_s}\left(1 - \frac{G_m}{G_s(\xi)}\right)} \\
\upsilon_{12} &= \upsilon_{12s}V_s + \upsilon_{12m}(1 - V_s) \\
\alpha_1 &= \frac{V_s\alpha_s E_s(\xi) + (1 - V_s)\alpha_m E_m}{E_{11}} \\
\alpha_2 &= \frac{E_m}{E_{22}}\left[\alpha_m\left(1 - \sqrt{V_s}\right) + \frac{\alpha_m\sqrt{V_s} - V_s(\alpha_m - \alpha_s)}{1 - \sqrt{V_s}\left(1 - \frac{E_m}{E_s(\xi)}\right)}\right] \\
G_s(\xi) &= \frac{E_s(\xi)}{2(1 + \upsilon_{12s})} \\
\rho &= \rho_s V_s + \rho_m(1 - V_s)
\end{aligned} \quad (4)$$

where the subscript 'm' and 's' stand for the metal substrate and the SMA fibers, respectively. Also, E, G, υ, α, and ρ represent Young's modulus, shear modulus, Poisson's ratio, thermal expansion coefficient, and material density, respectively.

In this work, the one-dimensional constitutive equation of SMA materials derived by Brinson [48] is used:

$$\sigma - \sigma_0 = E(\xi)\varepsilon - E(\xi_0)\varepsilon_0 + \Omega(\xi)\xi_S - \Omega(\xi_0)\xi_{S0} + \Theta(T - T_0) \quad (5)$$

where the subscript '0' indicates the parameter at the initial state. Moreover, σ,ε, ξ, $E(\xi)$, $\Omega(\xi)$ and Θ are stress, strain, martensite fraction, Young's modulus, transformation tensor, and thermoelastic tensor, respectively. The thermal expansion coefficient of SMA fibers is assumed as a function of the martensite volume fraction [49]. As a result, thermal expansion coefficient and Young's modulus are expressed by the Reuss approach [50]

$$\begin{aligned}
E_s(\xi) &= \frac{E_A}{1 + \left(\frac{E_A}{E_M} - 1\right)\xi} \\
\alpha_s(\xi) &= \frac{\alpha_A}{1 + \left(\frac{\alpha_A}{\alpha_M} - 1\right)\xi}
\end{aligned} \quad (6)$$

where subscripts 'A' and 'M' stand for the pure austenite and martensite phases, respectively. The phase transformation tensor is also assumed in terms of the Young's modulus of fibers as [48]