



Multi-site micromechanics of composite materials with temperature-dependent constituents under small strain and finite thermal perturbation assumptions



Yao Koutsawa*

Centre de Recherche Public Henri Tudor, 29, Avenue John F. Kennedy, L-1855 Luxembourg, Grand Duchy of Luxembourg

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ABSTRACT

This study presents mean-field based micromechanics models to predict the effective thermoelastic properties, namely, elasticities, thermal expansions and heat capacity, of thermoelastic composite materials with temperature-dependent constituents under finite temperature changes and small strain assumptions. First, the Helmholtz potential density for small strain finite thermoelasticity has been presented. The fundamental solution based on Green's function of elasticity problem has been used to derive the general expressions for the elastic and thermal strains concentration tensors with temperature-dependent material properties. A family of mean-field based micromechanics models (multi-site Eshelby dilute model, Mori–Tanaka model and self-consistent model) has been presented. The models are general enough to account for the morphology and topology textures of the microstructure of a thermoelastic composite. Numerical examples based on the multi-site Mori–Tanaka model are used to quantify the differences of the effective properties based on the small strain finite thermoelasticity in comparison to the linear thermoelasticity. The predictions of the multi-site Mori–Tanaka micromechanics model are also compared to those of the VAMUCH, a finite element based micromechanics model.

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1. Introduction

In many engineering applications, materials are often suggested to thermo-mechanical loading. In thermoelastic composite materials, significant thermal stresses can arise due to the mismatch in the coefficient of thermal expansions (CTEs) of the constituents, affecting their global performances. The use of composite materials in engineering structural components requires understanding the variations in the field variables (e.g., stress, strain, temperature, to name a few), both at the micro- and macro scales. To study the thermomechanical behavior of composite materials, the so-called linear thermoelasticity which, in addition to the temperature-independence, assumes that both the strains and the temperature relative variations are small, is often used. The small temperature relative variations implies that the temperature increment is small relative to the initial (reference) absolute temperature, T_0 . If the current temperature is T , the linear thermoelasticity requires $(T - T_0)/T_0$ to be of the order of the elastic strain which is in the order of 1% or smaller for it to be small. These

assumptions all together make the linear thermoelasticity theory highly constraining for engineering applications where the strain is small but the temperature relative variations are large and the material properties are temperature-dependent, see [1].

As, it is well stressed in [2], many engineering systems are designed to support significant thermal perturbations of hundreds or even thousands of degrees such as space shuttle thermal protection panels, gas turbine blades, car and airplane heat exchangers, to name a few. Since the CTEs for hard solids (e.g., diamonds, invar, silicon) and many metals and their alloys (e.g., steel, aluminium, copper) are in the order of $10^{-6}/^\circ\text{C}$ and $10^{-5}/^\circ\text{C}$, respectively, the strains arise in most engineering materials can still be reasonably assumed small even if the temperature variations are large. It is therefore obvious, for practical reasons, to abandon the small thermal perturbations and temperature independence of the material properties assumptions without violating the small strain assumption.

Different types of mean-field based micromechanics approaches have been proposed to predict the effective elastic and/or thermal properties of composite materials. The self-consistent approach [3], the generalized self-consistent scheme [4], the Mori–Tanaka model [5], the differential method [6] are some examples. Fundamentals of these micromechanics models can be

* Tel.: +352 42 59 91 48 79; fax: +352 42 59 91 555.

E-mail address: yao.koutsawa@tudor.lu

found, for e.g., in the monograph [7]. Micromechanics modeling of the thermoelastic behavior of composite materials is usually addressed using the linear thermoelasticity, see, for e.g., [8–11] and references cited therein. The case of composites with temperature-dependent constituents properties has been less investigated. As it has been said before, the two assumptions of the linear thermoelasticity, namely, the small temperature variations and temperature-independence of the material properties, are too restrictive to be useful for many practical engineering applications. Hence, in the vein of the work reported in [2], the objective of this work is to relax these two assumptions by extending the multi-site mean-field based micromechanics model presented in [11] to thermoelastic composite materials made of temperature-dependent constituents and subject to finite temperature changes.

The multi-site mean-field based micromechanics model reported in [11] is versatile enough to handle relatively complex microstructures. It is able to take into account the morphology and topology textures of the reinforcements. Hence, its extension for thermoelastic composite materials made of temperature-dependent constituents and subject to finite temperature changes may be useful for handling particle, short fibers and long fibers reinforced thermoelastic composite materials in practical engineering material systems. This paper is organized as follows. The small strain finite thermoelasticity framework is presented in Section 2 where the terminology and notation as well as the expressions for the Helmholtz potential of a homogeneous thermoelastic material are given. The localization relations combined with the local Helmholtz potential give the macroscopic Helmholtz potential of a heterogeneous thermoelastic material. The effective thermoelastic properties are then deduced from the macroscopic Helmholtz potential. Section 3 presents the integral equations and the explicit expressions for the elastic and thermal strains concentration tensors in the case of the multi-site micromechanics for small strain finite thermoelasticity. To validate the proposed mean-field based micromechanics model, Section 4 presents the variational asymptotic method for unit-cell homogenization (VAMUCH), a finite element based micromechanics model, see [2,12], for the small strain finite thermoelasticity. Numerical examples are given in Section 5 to quantify the difference of the results based on the small strain finite thermoelasticity in comparison to the linear thermoelasticity. A summary of the research findings is given in Section 6.

2. Fundamentals of micromechanics for small strain finite thermoelasticity

2.1. Terminology and notation

The material thermoelastic properties of interest in this study are the isothermal stiffness tensor, \mathbf{L} , the thermal expansion tensor, α , and the heat capacity per unit of volume at constant strain, C_ϵ . These properties are defined as follows

$$\mathbf{L}(T) \equiv \left(\frac{\partial \sigma}{\partial \epsilon} \right)_T, \quad (1)$$

$$\alpha(\sigma, T) \equiv \left(\frac{\partial \epsilon}{\partial T} \right)_\sigma, \quad (2)$$

$$C_\epsilon(\epsilon, T) \equiv T \left(\frac{\partial \delta}{\partial T} \right)_\epsilon. \quad (3)$$

In the above definitions, the symbol, \equiv , denotes equality by definition, σ is the stress tensor, ϵ is the strain tensor, T is the absolute temperature, and η is the entropy. A property, P , will be denoted by either $P(\epsilon, T)$ or $P(\sigma, T)$. P_0 is the value of P in the reference state, which is assumed here to be the natural state ($T = T_0$, $\epsilon = \mathbf{0}$, $\sigma = \mathbf{0}$). The thermal expansion defined in Eq. (2) is

in line with thermodynamic conventions. Usually within the small strain thermoelasticity, two alternative measures are used: the instantaneous or tangent thermal expansion, $\alpha_{\text{tan}}(T)$, defined by

$$\alpha_{\text{tan}}(T) \equiv \alpha(\sigma = \mathbf{0}, T), \quad (4)$$

and the secant thermal expansion, $\alpha_{\text{sec}}(T)$, defined by

$$\alpha_{\text{sec}}(T) \equiv \frac{1}{T - T_0} \int_{T_0}^T \alpha(\sigma = \mathbf{0}, v) dv = \frac{1}{T - T_0} \int_{T_0}^T \alpha_{\text{tan}}(v) dv, \quad (5)$$

where T_0 is the reference (initial) temperature. $\alpha_{\text{tan}}(T)$ and $\alpha_{\text{sec}}(T)$ are the *free* thermal expansions, i.e. thermal expansions under zero stress conditions.

In what follows, the dot “ \cdot ” denotes the dot product between second-order tensors, the colon “ $:$ ” denotes the double dot product between fourth-order tensor and second (or fourth)-order tensor and $\theta = T - T_0$ denotes the temperature perturbation.

2.2. Helmholtz potential of a homogeneous thermoelastic material

The expression for the Helmholtz potential density, $f(\epsilon, T)$, of a homogeneous thermoelastic material under small strain and finite thermal perturbation assumptions is given in [13] as follows

$$f(\epsilon, T) = \frac{1}{2} \epsilon \cdot \mathbf{L}(T) : \epsilon + \lambda(T) \cdot \epsilon - \int_{T_0}^T \left(\int_{T_0}^\zeta \frac{C_\epsilon(\epsilon = \mathbf{0}, v)}{v} dv \right) d\zeta - \eta_0(T - T_0) + f_0, \quad (6)$$

where

$$\lambda(T) = -\mathbf{L}(T) : \int_{T_0}^T \alpha_{\text{tan}}(v) dv = -\mathbf{L}(T) : \alpha_{\text{sec}}(T) \theta. \quad (7)$$

The state equations corresponding to this potential, read

$$\sigma(\epsilon, T) = \frac{\partial f}{\partial \epsilon}(\epsilon, T) = \mathbf{L}(T) : \epsilon + \lambda(T), \quad (8)$$

$$\eta(\epsilon, T) = -\frac{\partial f}{\partial T}(\epsilon, T) = -\frac{1}{2} \epsilon \cdot \frac{d\mathbf{L}}{dT} : \epsilon - \frac{d\lambda}{dT} \cdot \epsilon + \int_{T_0}^T \frac{C_\epsilon(\epsilon = \mathbf{0}, v)}{v} dv + \eta_0, \quad (9)$$

$$C_\epsilon(\epsilon, T) = -T \frac{\partial^2 f}{\partial T^2}(\epsilon, T) = C_\epsilon(\epsilon = \mathbf{0}, T) - T \left(\frac{1}{2} \epsilon \cdot \frac{d^2 \mathbf{L}}{dT^2} : \epsilon + \frac{d^2 \lambda}{dT^2} \cdot \epsilon \right). \quad (10)$$

2.3. Localization relations

Representative volume element (RVE) under controlled macroscopic (average) strain, $\bar{\epsilon}$, and temperature, T , is considered. The RVE consists of reinforcements embedded in a continuous material. The domain of the RVE is denoted by V . The localization relations or obtained by solving the following linear elasticity problem, $\mathcal{P}(\mathbf{L}, \lambda, \bar{\epsilon}, V)$, defined as: given $(\mathbf{L}, \lambda, \bar{\epsilon}, V)$ find $(\mathbf{u}, \epsilon, \sigma)$ such that

$$\text{div } \sigma = \mathbf{0} \quad \text{in } V, \quad (11a)$$

$$\sigma = \mathbf{L} : \epsilon + \lambda, \quad \text{in } V, \quad (11b)$$

$$\epsilon = \frac{1}{2} [\text{grad } \mathbf{u} + (\text{grad } \mathbf{u})^\top] \quad \text{in } V, \quad (11c)$$

$$\mathbf{u} = \bar{\epsilon} \mathbf{x} \quad \text{on } \partial V, \quad (11d)$$

where div and grad are the divergence and the gradient operators, respectively; \top is transposition; both \mathbf{L} and λ are functions of the position, \mathbf{x} , and the temperature, T , whereas $\bar{\epsilon}$ is position-independent. It is known that the average of ϵ over V is equal to $\bar{\epsilon}$,

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