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Fully coupled electromechanical buckling analysis of active laminated composite plates considering stored voltage in actuators

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ABSTRACT

In practice, voltage would be stored in actuators due to applied mechanical loads or voltage specified as input to them. The effect of stored voltage in actuators on buckling analysis has not been studied so far. This article addresses fully coupled electromechanical buckling analysis of active laminated composite thin to thick plates using partial hierarchical Rayleigh–Ritz solution. The formulation is derived from variational principle with consideration for shear deformation plate theory. Another major contribution is to postpone the initiation of buckling of a laminated plate by increasing its flexural stiffness using feedback piezoelectric control.

1. Introduction

In recent years, the application of piezoelectric material has been developing to enhance the performance of engineering structures in multi-functional purposes. They could be used as sensors and actuators for monitoring and controlling the behavior of structures, respectively. Because of having the ability to sense and react simultaneously, sensor and actuator pairs as smart systems are used to control authority over their static/dynamic responses for complicated applications. Additionally, advantages of composite materials have greatly increased their popularity in engineering applications. The collocation of composite structures as passive ones with the smart systems leads to create smart structures which have attracted the attention of researchers to gain a better understanding of their mechanical behavior.

Buckling phenomenon is a mode of failure which a structure can experience in certain situations. Therefore, studying the active buckling control may be effective to monitor and control the structure deflection in cases where structures need to carry more loads.

Intensive research activities have been carried in development of laminated composite piezoelectric plates [1–4] by many researchers. Numerous buckling studies for piezoelectric laminated plates subjected to mechanical, thermal or electrical loading are available in the literature. Oh et al. [5] studied the post-buckling and vibration behaviors of piezo-laminated plates under thermal-electric effects, using finite element formulation based on a layer-wise theory. Shen [6,7] analyzed the buckling and post-buckling behaviors of imperfect laminated plates with piezoelectric actuators under different electro-thermo-mechanical loading. Varelis and Saravanos presented a finite element model for active buckling [8] and post-buckling [9] of composite beams/plates with piezoelectric actuators, under electromechanical loading. Kapuria and Achary [10–12] developed exact 3*D* and 2*D* piezoelasticity solution for buckling of hybrid piezoelectric plates under in-plane electro-thermo-mechanical loading, based on zigzag theory. Giannopoulos et al. [13] employed finite element formulation and experimental method to present buckling behavior of smart beams/plates under electro-thermo-mechanical loading. Kim and Lee [14] presented an exact 3*D* solution for buckling of a piezoelectric laminate with weak interfaces. Akhras and Li [15] presented a 3*D* analysis of statics, vibrations, and stability of piezoelectric plates using a finite layer approach. Then, they employed this method for 3*D* thermal buckling analysis of symmetrical [16] and anti-symmetrical [17] piezoelectric laminated plates under electro-thermo-mechanical loading. Moreover, they [18] studied free vibration and stability analysis for piezoelectric composite plates under mechanical load using a spline finite strip.

The flexural rigidity and load-carrying capability of a plate can be increased by active methods. Limited works have been reported on the control buckling of laminated plates using piezoelectric properties.

Meressi and Paden [19], as well as Thompson and Loughlan [20] employed piezoelectric actuators to control buckling of beams/columns by applying a controlled voltage to actuators. Chandrashekhara and Bhatia [21] presented a finite-element model for the dynamic buckling control of smart plates under a linearly increasing compressive load. Birman [22] introduced the active control of stability and vibration for composite plates using piezoelectric

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stiffeners under static and dynamic electric fields. Batra and Geng [23] developed a finite element model to enhance the dynamic buckling load for a plate under compressive variable load by using piezoceramic actuators. Shariyat presented a finite element formulation to control the vibration and dynamic buckling of laminated [24] and FGM [25] plates by piezoelectric sensor/actuator pairs under thermo-electro-mechanical loading.

Although few works [5,13,26,27] have been done on studying the buckling of piezoelectric plates under different simultaneous thermal, electrical, and mechanical loads, the effect of produced voltage in actuators did not investigated. In practice, applied mechanical and thermal loads, and voltages specified as input to the actuators cause a voltage to be produced in them. This produced voltage would be stored in actuators. To our knowledge only one work [28] has been carried out on piezoelectric structures based on fully coupled electromechanical loading. Ha et al. [28] presented a finite element model for static and vibration analysis of composite structures containing distributed piezoceramic sensors and actuators with consideration for stored voltage in actuators. However, they did not study buckling analysis.

Therefore, the objective of this work is to study the stored voltage in actuators and its effects on buckling of laminated plates. Another objective of present work is to actively control static buckling of piezoelectrically-stiffened laminated thick and moderately thick plates by integrating piezoelectric transduction layers, which has not been accomplished yet.

This article deals with solving fully coupled electromechanical buckling for laminated composite thin to thick active plates using partial hierarchical Rayleigh–Ritz solution. Furthermore, active control of laminated composite plates with actuator/sensor pairs under a compressive load is studied using feedback control law. Using a subset of partial hierarchical shape functions, we develop partial hierarchical Rayleigh–Ritz (PHRR) method [29,30] based on von Karman's moderate rotation kinematics and according to CLPT, FSDT, and TSDT kinematical models, to derive the nonlinear governing equations of buckling phenomenon. Then, we investigate the effects of different electrical boundary conditions, as well as the effects of piezoelectric parameters, such as voltage control signal and feedback gain on the critical load.

2. Governing equations

The geometric parameters of the plate and coordinate system are shown in Fig. 1. The geometry includes length a, width b, and total thickness h. The plate is composed of a multilayered composite substrate and two piezoelectric layers attached to the both sides. In Fig. 1, h_a and h_s are thickness of actuator and sensor layers, respectively.

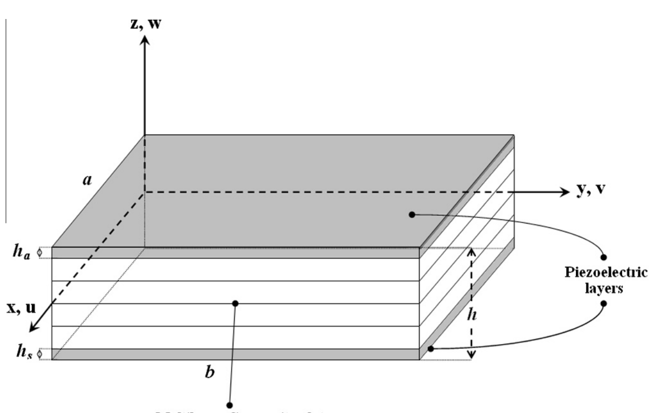

Fig. 1. The geometric parameters of the plate and the coordinate system.

2.1. Kinematic relations

In the present analysis, the mixed-field laminate plate theory [8] is developed using Reddy's TSDT [31]. Therefore, displacement field and electric potential are written as:

$$
\begin{aligned}
u(x,y,z) &= u_0(x,y) + z\beta_x(x,y) - c_1 z^3\left(\beta_x + \frac{\partial w_0}{\partial x}\right)\\
v(x,y,z) &= v_0(x,y) + z\beta_y(x,y) - c_1 z^3\left(\beta_y + \frac{\partial w_0}{\partial y}\right)\\
w(x,y,z) &= w_0(x,y)\\
\varphi(x,y,z) &= \sum_{l=1}^{N}\varphi_l(x,y)\psi_l(z)
\end{aligned} \tag{1}
$$

where u_0, v_0, w_0, β_x, and β_y are the translational and the rotational displacement components of mid-surface of the plate, φ is the electric potential field for any generic point, φ_l are electric potential values according to layerwise electric potential field [8], ψ_l are linear interpolation functions, and finally c_1 is $4/3h^2$. The displacement and the rotation components of mid-surface can be written in a vector form as:

$$
u_0 = \mathbf{N_u}\hat{\mathbf{u}}, \quad v_0 = \mathbf{N_v}\hat{\mathbf{v}}, \quad w_0 = \mathbf{N_w}\hat{\mathbf{w}}, \quad \beta_x = \mathbf{N_\beta}\hat{\boldsymbol{\beta}}_\mathbf{x}, \quad \beta_y = \mathbf{N_\beta}\hat{\boldsymbol{\beta}}_\mathbf{y} \tag{2}
$$

where $\mathbf{N_u}$, $\mathbf{N_v}$, $\mathbf{N_\beta}$, and $\mathbf{N_w}$ are polynomial shape function based on hierarchical functions and Pascal's polynomials [32,33], respectively, and $\hat{\mathbf{u}}$, $\hat{\mathbf{v}}$, $\hat{\mathbf{w}}$, $\hat{\boldsymbol{\beta}}_\mathbf{x}$, and $\hat{\boldsymbol{\beta}}_\mathbf{y}$ are Rayleigh–Ritz coefficients vector. Therefore, the mid-surface displacements and the rotations can be rewritten as a vector:

$$
\begin{aligned}
&\mathbf{U} = \mathbf{N_R}\hat{\mathbf{U}}\\
&\mathbf{N_R} = [\mathbf{N_u} \quad \mathbf{N_v} \quad \mathbf{N_w} \quad \mathbf{N_\beta} \quad \mathbf{N_\beta}]\\
&\hat{\mathbf{U}}^\mathbf{T} = \left[\hat{\mathbf{u}}^\mathbf{T} \quad \hat{\mathbf{v}}^\mathbf{T} \quad \hat{\mathbf{w}}^\mathbf{T} \quad \hat{\boldsymbol{\beta}}_\mathbf{x}^\mathbf{T} \quad \hat{\boldsymbol{\beta}}_\mathbf{y}{}^\mathbf{T}\right]
\end{aligned} \tag{3}
$$

The von-Karman's strains are expressed as [31]:

$$
\begin{aligned}
&\varepsilon_{xx} = \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2, \quad \varepsilon_{yy} = \frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2,\\
&\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \left(\frac{\partial w}{\partial x}\right)\left(\frac{\partial w}{\partial y}\right)\\
&\gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}, \quad \gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}
\end{aligned} \tag{4}
$$

The strain–displacement relations can be rewritten in the vector notation as:

$$
\begin{aligned}
&\bar{\boldsymbol{\varepsilon}} = \bar{\boldsymbol{\varepsilon}}^{(\mathbf{01})} + \bar{\boldsymbol{\varepsilon}}^{(\mathbf{02})} + z\bar{\boldsymbol{\varepsilon}}^{(\mathbf{1})} + z^3\bar{\boldsymbol{\varepsilon}}^{(\mathbf{3})}, \quad \bar{\boldsymbol{\varepsilon}} = [\varepsilon_{xx} \quad \varepsilon_{yy} \quad \gamma_{xy}]\\
&\bar{\boldsymbol{\gamma}} = \bar{\boldsymbol{\gamma}}^{(\mathbf{0})} + z^2\bar{\boldsymbol{\gamma}}^{(\mathbf{2})}, \quad \bar{\boldsymbol{\gamma}} = [\gamma_{yz} \quad \gamma_{xz}]
\end{aligned} \tag{5}
$$

where

$$
\begin{aligned}
&\bar{\boldsymbol{\varepsilon}}^{(\mathbf{01})} = \begin{Bmatrix} \frac{\partial u_0}{\partial x} \\ \frac{\partial v_0}{\partial y} \\ \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} \end{Bmatrix}, \quad \bar{\boldsymbol{\gamma}}^{(\mathbf{0})} = \begin{Bmatrix} \beta_y + \frac{\partial w_0}{\partial y} \\ \beta_x + \frac{\partial w_0}{\partial x} \end{Bmatrix}, \quad \bar{\boldsymbol{\varepsilon}}^{(\mathbf{02})} = \begin{Bmatrix} \frac{1}{2}\left(\frac{\partial w_0}{\partial x}\right)^2 \\ \frac{1}{2}\left(\frac{\partial w_0}{\partial y}\right)^2 \\ \frac{\partial w_0}{\partial x}\frac{\partial w_0}{\partial y} \end{Bmatrix}\\
&\bar{\boldsymbol{\varepsilon}}^{(\mathbf{1})} = \begin{Bmatrix} \frac{\partial \beta_x}{\partial x} \\ \frac{\partial \beta_y}{\partial y} \\ \frac{\partial \beta_x}{\partial y} + \frac{\partial \beta_y}{\partial x} \end{Bmatrix}, \quad \bar{\boldsymbol{\varepsilon}}^{(\mathbf{2})} = -3c_1\begin{Bmatrix} \beta_y + \frac{\partial w_0}{\partial y} \\ \beta_x + \frac{\partial w_0}{\partial x} \end{Bmatrix},\\
&\bar{\boldsymbol{\varepsilon}}^{(\mathbf{3})} = -c_1\begin{Bmatrix} \frac{\partial \beta_x}{\partial x} + \frac{\partial^2 w_0}{\partial x^2} \\ \frac{\partial \beta_y}{\partial y} + \frac{\partial^2 w_0}{\partial y^2} \\ \frac{\partial \beta_x}{\partial y} + \frac{\partial \beta_y}{\partial x} + 2\frac{\partial^2 w_0}{\partial x \partial y} \end{Bmatrix}
\end{aligned} \tag{6}
$$