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Surface effects on the nonlinear forced vibration response of third-order shear deformable nanobeams

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ABSTRACT

Given the high surface to volume ratio, the nonlinear forced vibration behavior of third-order shear deformable nanobeams in the presence of the both effects of surface stress and that of surface inertia is investigated. Gurtin–Murdoch elasticity theory is utilized within the framework of third-order shear deformation beam theory to develop a novel non-classical beam model to incorporate surface effects into the forced vibration analysis of nanobeams. A cubic variation through the thickness of nanobeam is considered for the normal stress component of the bulk in order to satisfy the surface equilibrium equations. Hamilton's principle is used to derive size-dependent nonlinear governing differential equations of motion. The equations are solved numerically using generalized differential quadrature method with an iterative algorithm on the basis of shifted Chebyshev–Gauss–Lobatto grid points. Subsequently, based on the Galerkin's technique, the set of nonlinear partial differential equations are reduced into a time-varying set of ordinary differential equations of Duffing type. At the end, the pseudo arc-length method is employed to solve the set of nonlinear equations of the time domain. It is observed that by increasing the beam thickness, surface effects on the nonlinear forced vibration behavior of nanobeam diminish which leads to increasing the deviation from the linear response.

1. Introduction

The investigations about mechanical characteristics of micro- and nano-sized structures such as nanobeams are major topic of current interest by researchers. Nowadays, beam structures are widely used in various systems at micro and nano-scale [1–6]. Time-dependent external forces are usual in dynamic systems that cause to the forced vibration in which the amplitude of system depends on the frequency ratio. Moreover, it has been experimentally indicated that the behaviors of structures at micron and sub-micron scales are size-dependent [7–9]. The classical continuum mechanics does not have the capability to take size effects into account, so in order to interpret the size-dependent responses of nanostructures, different non-classical continuum theories have been introduced and employed during past years [10–21].

One of the main factors in size-dependency of nanoscale behaviors is surface effect. Because of the high ratio of surface area to bulk volume, the surface of nanostructures has a substantial influence on the mechanical characteristics. Gurtin and Murdoch [22,23] developed a theoretical framework based on the continuum mechanics including surface effects which has an excellent capability to incorporate the surface effects into the mechanical response of nanostructures. According to this theory, the surface can be actually modeled as an elastic membrane without thickness which is perfectly bonded to the bulk of structure. Many efforts have been made to consider surface effects on different mechanical responses of nanostructures based on Gurtin–Murdoch surface elasticity theory.

Li et al. [24] studied the influence of surface effect on stress concentration around a spherical cavity in a linearly isotropic elastic medium on the basis of continuum surface elasticity. Mogilevskaya et al. [25] considered a two-dimensional problem of multiple interacting circular nano-inhomogeneities and nano-pores based on Gurtin–Murdoch model. Luo and Wang [26] investigated the elastic field of an elliptic nano inhomogeneity embedded in an infinite matrix under anti-plane shear. The interface stress effects of the nano inhomogeneity were accounted for with Gurtin–Murdoch model. Gordeliy et al. [27] analyzed a two-dimensional, transient, uncoupled thermoelastic problem of an infinite medium with a circular nano-scale cavity using Gurtin–Murdoch elasticity theory. Zhao and Rajapakse [28] applied the Gurtin–Murdoch continuum model accounting for surface energy effects on the elastic field of an isotropic elastic layer bonded to a rigid base. Song and Huang

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[29] applied the incremental deformation theory to study the surface stress effects upon the bending behaviors of nanowires on the basis of Gurtin–Murdoch elasticity theory. Mogilevskaya et al. [30] studied the effects of surface elasticity and surface tension on the transverse overall behavior of unidirectional nanoscale fiber-reinforced composites. They regarded the interfaces between the nano-fibers and the matrix as material surfaces described by the Gurtin–Murdoch model. Ansari and Sahmani [31] proposed a non-classical solution to analyze the bending and buckling responses of nanobeams including surface stress effect. They implemented the Gurtin–Murdoch elasticity theory into the various classical beam theories. Also, Ansari and Sahmani [32] investigated the free vibration characteristics of nanoplates incorporating the effect of surface stress using surface continuum elasticity.

To mention some more recent studies, Zhao and Rajapakse [33] considered the general three-dimensional asymmetric problem for an elastic layer of nanoscale thickness that is bonded to a rigid substrate and subjected to tangential loading at the surface using Gurtin–Murdoch surface elasticity. Ansari et al. [34] performed a numerical analysis on the postbuckling response of nanobeams with the consideration of the surface stress effect. They applied the Gurtin–Murdoch elasticity theory to the classical Euler–Bernoulli beam theory to develop non-classical beam model. Also, they conducted the same attempt for Timoshenko nanobeams [35]. Shaat et al. [36] developed a new Kirchhoff plate model using a modified couple-stress theory to investigate the bending behavior of nanoplates. They used the surface elasticity theory of Gurtin–Murdoch to model the surface energy effects into the framework of the modified couple stress theory of elasticity.

In the present investigation, the nonlinear forced vibration response of third-order shear deformable nanobeams is predicted in the presence of surface effects. Gurtin–Murdoch continuum elasticity is applied to the classical third-order shear deformation beam theory to develop size-dependent beam model. In order to satisfy the balance conditions on the surfaces of nanobeam, it is assumed that the normal stress component of the bulk is distributed cubically through the thickness. After that, the variational technique on the basis of Hamilton's principle is utilized to derive non-classical governing differential equations of motion and associated boundary conditions. Generalized differential quadrature (GDQ) method with an iterative algorithm on the basis of shifted Chebyshev–Gauss–Lobatto grid points is then employed to discretize governing equations. Galerkin's approach is used to reduce the set of nonlinear equations into a time-varying set of ordinary differential equations of Duffing type.

2. Mathematical formulations

Among various types of the classical beam theory, in the third-order shear deformation beam theory, there is no shear correction factor to estimate the distribution of shear strain across beam thickness. In this theory, it is assumed that the transverse shear strains are assumed to be distributed parabolically through the beam thickness as shown in Fig. 1. According to this type of beam theory, the components of displacement vector for an arbitrary point can be defined as

$$u_x = U(x,t) + z\Psi(x,t) - \frac{4z^3}{3h^2}\left(\Psi(x,t) + \frac{\partial W(x,t)}{\partial x}\right),$$
$$u_y = 0, \quad u_z = W(x,t) \tag{1}$$

in which $U(x,t)$, $W(x,t)$ and $\Psi(x,t)$ represent, respectively, the axial displacement of the center of sections, the lateral deflection of the beam, and the rotation angle of the cross section with respect to the vertical direction.

In the current study, it is assumed that the slops in the beam after deformation are very small. Therefore, the strains of a

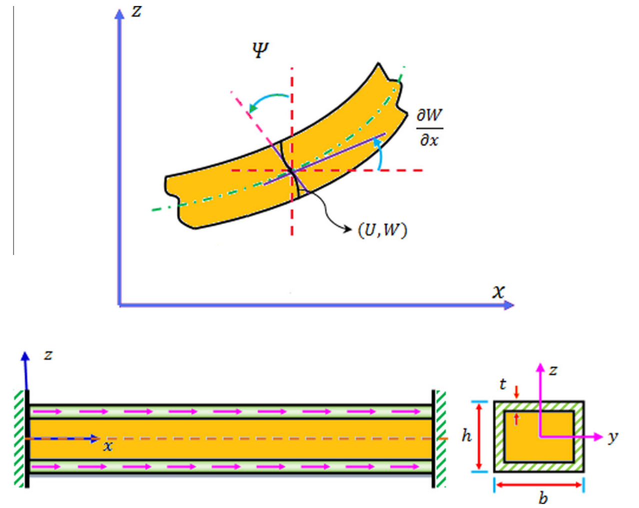

Fig. 1. Schematic view of a third-order shear deformable nanobeam with the kinematic parameters and coordinate system.

third-order shear deformable nanobeam can be approximated by the von-Karman relation as

$$\varepsilon_{xx} = \frac{\partial u_x}{\partial x} + \frac{1}{2}\left(\frac{\partial W}{\partial x}\right)^2$$
$$= \frac{\partial U}{\partial x} + z\frac{\partial \Psi}{\partial x} - \frac{4z^3}{3h^2}\left(\frac{\partial \Psi}{\partial x} + \frac{\partial^2 W}{\partial x^2}\right) + \frac{1}{2}\left(\frac{\partial W}{\partial x}\right)^2 \tag{2a}$$

$$\varepsilon_{xz} = \frac{1}{2}\left(\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x}\right) = \frac{1}{2}\left(\frac{\partial W}{\partial x} + \Psi\right) - \frac{2z^2}{h^2}\left(\Psi + \frac{\partial W}{\partial x}\right) \tag{2b}$$

On the basis of the linear elasticity, the non-zero stress components for the nanobeam can be introduced as

$$\sigma_{xx} = (\lambda + 2\mu)\left[\frac{\partial U}{\partial x} + z\frac{\partial \Psi}{\partial x} - \frac{4z^3}{3h^2}\left(\frac{\partial \Psi}{\partial x} + \frac{\partial^2 W}{\partial x^2}\right) + \frac{1}{2}\left(\frac{\partial W}{\partial x}\right)^2\right] \tag{3a}$$

$$\sigma_{xz} = \mu\left[\left(1 - \frac{4z^2}{h^2}\right)\left(\Psi + \frac{\partial W}{\partial x}\right)\right] \tag{3b}$$

where $\lambda = \frac{\nu E}{(1+\nu)(1-2\nu)}$ and $\mu = \frac{E}{2(1+\nu)}$. Also, E and ν denote the Young's modulus and Poisson's ratio, respectively.

Based on the Gurtin–Murdoch elasticity theory, the following general and simple expressions for surface stress–strain relation can be defined as [22,23]

$$\sigma^s_{\alpha\beta} = \tau_s\delta_{\alpha\beta} + (\tau_s + \lambda_s)\varepsilon_{\gamma\gamma}\delta_{\alpha\beta} + 2(\mu_s - \tau_s)\varepsilon_{\alpha\beta} + \tau_s u^s_{\alpha,\beta} \tag{4a}$$

$$\sigma^s_{\alpha z} = \tau_s u^s_{z,\alpha} \tag{4b}$$

in which λ_s and μ_s are the surface Lame constants and τ_s is the residual surface stress under unstrained conditions.

Now, by using the above equations, the non-zero components of surface stress can be obtained for a third-order shear deformable nanobeam as

$$\sigma^s_{xx} = (\lambda_s + 2\mu_s)\left[\frac{\partial U}{\partial x} + z\frac{\partial \Psi}{\partial x} - \frac{4z^3}{3h^2}\left(\frac{\partial \Psi}{\partial x} + \frac{\partial^2 W}{\partial x^2}\right) + \frac{1}{2}\left(\frac{\partial W}{\partial x}\right)^2\right]$$
$$- \frac{\tau_s}{2}\left(\frac{\partial W}{\partial x}\right)^2 + \tau_s \tag{5a}$$