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Transverse orthotropic elastic moduli of bundled coated thin elliptical tubes

Tejas P. Gotkhindi *, K.R.Y. Simha

Department of Mechanical Engineering, Indian Institute of Science, Bangalore 560012, India

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ABSTRACT

Light weight structures with tailored mechanical properties have evolved beyond regular hexagonal/circular honeycomb topology. For applications which demand anisotropic mechanical properties, elliptical-celled structures offer interesting features. This paper characterizes the anisotropic in-plane elastic response of coated thin elliptical tubes in different array patterns viz. close-packed, diagonal and rectangular patterns under compression. This paper also extends earlier works on elliptical close-packed structure to a more general case of coated tubes. Theoretical framework using thin ring theory provides formulae in terms of geometric and material parameters. These are compared with a series of FE simulations using contact elements. The FE results are presented as graphs to aid in design.

1. Introduction

Tube bundles for structural load bearing pervade in nature as vascular bundles in woods. Large axial-to-transverse length ratios offer flexibility in transverse direction with relatively high stiffness in the axial direction. In man-made applications bundled tubes play a primary function to conduct fluids as in heat exchangers and in off-shore platforms. Bundled tubes with low axial-to-transverse length ratios constitute a type of cellular solid. These are extensively used in applications like energy absorption systems and for cores in sandwich composites. These applications require overall mechanical properties to characterize structural integrity and in particular, the in-plane properties as they are largely loaded in the transverse direction.

Mechanical response of these structures either enhances or deteriorates when they are deliberately or inadvertently reinforced by a secondary material. Former is achieved by applying coats and by use of liners while latter results from corrosion scale deposits. Coatings primarily protect the substrate from corrosive environment while liners add strength to the structure. Scales, on the other hand, build on substrate due to corrosive environment.

Extensive literature on mechanical properties of cellular solids like regular hexagonal, square and triangular honeycomb structure has been reported [1]. Mechanical properties of regular hexagonal cellular solids are sensitive to imperfections and circular honeycomb structures offer an alternative structure [2–4]. However, their inherent isotropy limits their applications to cases with directional anisotropic requirement.

Elliptical-celled honeycomb structures have been proposed and used to cater these requirements [2,5]. Elliptical cellular structures have been comparatively less explored compared to other cellular solids. In particular, only close-packed (quasi-hexagonal) elliptical cellular solids have been explored. Reinforced or coated thin elliptical *tubes* and elliptical cellular structures in other topologies like rectangular and diagonal configurations have not been explored.

Chung and Waas [5] presented in-plane elastic moduli of elliptical and circular honeycomb structures. They characterized elastic response by evaluating Young's modulus, Poisson's ratio and shear modulus using Castigliano's theorem by considering only flexural energy. They used two term approximation for differential arc length and gave expressions for moduli in terms of ellipticity ratio valid near circular shape. Recently, they have characterized the same using couple-stress theory [6]. Lin and Huang [2] extended the analysis of Chung and Waas [5] by involving axial and shear deformations, and thickness. They gave expression for Young's moduli and Poisson's ratios obtained by using theoretical analysis data in conjunction with regression analysis. They also discussed about plastic yielding, brittle crushing strength and recently explored creep-rupturing of such structures [7]. The equivalent elastic modulus of an alveolar cell has been explored by modeling it as a thin elliptical ring filled with elastic medium [8].

This paper explores the transverse effective moduli of coated thin elliptical cellular solids/bundled tubes in different topologies – rectangular and diagonal. These configurations can model wood

* Corresponding author. Tel.: +91 9449850748.
E-mail addresses: tejas@mecheng.iisc.ernet.in (T.P. Gotkhindi), simha@mecheng.iisc.ernet.in (K.R.Y. Simha).

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Nomenclature

a,b	semi-major, semi-minor axis
ds	differential arc length
$E_A/E_B, \nu$	Young's modulus of coating/substrate
F_x/F_y	force in X/Y direction
E, G, I	parent material's Young's modulus, shear modulus, moment of inertia
E_0	substrate materials Young's modulus
$\widehat{EI}$	flexural rigidity
$E_y/E_x, G_{xy}$	Young' modulus in Y-direction/X-direction, shear modulus.
F	nodal force
$H/P/Q/H_1/H_2$	horizontal force
M/M_1	moments at section
$t_A/t_B, t$	thickness of coating/substrate, total thickness
u_x/u_y	displacement in X-direction/Y-direction.
w, υ	radial/tangential displacement
$w'''\cdots$	derivatives of w
w_d/w_e	radial displacement at Points d/e
$V/V_1/V_2$	vertical force
U_x^f/U_x^e	displacement in X-direction at point f/e
$U_{total-1}/U_{total-2}$	energy stored in integration limits
U_{total}/U'_{total}	total energy/total of flexural and axial energy.
$\bar{y}$	distance of neutral axis from the bottom fiber.
α	ellipticity ratio $(a/b-1)$
ϵ_y/ϵ_x	strain in Y/X direction.
ϵ_{xy}, τ	shear strain/shear stress
ϵ_θ	tangential strain
γ	angle subtended by tangent with horizontal
κ	bulk modulus
ν_{xy}, ν_{yx}	major Poisson's ratio/minor Poisson's ratio
φ	polar angle of point on the ring
ρ, ρ_0	Final radius of curvature/initial radius of curvature
θ	parametric angle
$\sigma_y/\sigma_a/\sigma_f$	far-field stress

Acronyms

FBD	free body diagram
RUC	representative unit cell
FEA	finite element analysis

as a bundle of elliptical tubes to represent orthotropy. We also refine the expressions for moduli of close-packed array and extend their range of applicability. The results are corroborated with FE simulations incorporating contact analysis. Applications of these have been highlighted by evaluating transverse bulk modulus along with an outline to extend the analysis to general configurations.

2. Analysis using thin ring theory

A set of straight beams is used to model and analyze curved structure like thin arches and rings [9]. The same model has been extended for the analysis of an elliptical hexagonal honeycomb structure [2,5]. Energy methods like Castigliano's theorems offer an easy and a convenient method to analyze any arbitrary shaped ring compared to the direct integration of governing differential equations which are linear ODEs with variable coefficients. However, for a circular ring the two methods are equally effective as the resulting governing differential equations are linear ODEs with constant coefficients [10,11]. Here, we adopt the energy method to analyze the coated bundled elliptical tubes (Plane Strain) and coated elliptical honeycomb structures (Plane Stress) in hexagonal, rectangular and diagonal arrays. The analysis presented in the following sub-sections is for an elliptical honeycomb structure (Plane Stress) but can be equally applied to a ring of any arbitrary shape. The extension to bundled elliptical tubes (Plane Strain) can be effected by substituting E by $E/(1-\nu^2)$.

The analysis assumes a large number of thin elliptical rings connected at the point of contacts through weld [2]. This assumption helps in reducing the analysis to a ring with point loads at the contact points. Twofold symmetry, further, reduces the analysis to a quarter of the ring. Internal moments and forces on the ring are assumed to be per unit depth and equal across the depth. The parent material is linearly elastic and the structure is assumed to exhibit linear trend between the applied loads and the corresponding displacements. These conditions suffice for invoking Castigliano's theorems.

As shown in Fig. 1, the ellipticity ratio $\alpha = a/b - 1$ [2] is used to characterize the geometry. Coating's thickness is t_A with elastic moduli E_A and ν while the substrate's thickness is t_B with properties E_B and ν. The modulus mismatch is restricted to Young's modulus while Poisson's ratios of the coating and the substrate are same. The total thickness is $t = t_A + t_B$ with bonding assumed perfect between the coating and the substrate.

The following two subsections relate the structures to moduli and lay an outline for the analysis with an emphasis on the arc length approximation. The third subsection details the analysis for the case of hexagonal array under uniaxial loading in Y-direction. Subsequent subsections present the results with brief descriptions.

2.1. Structural symmetry and number of elastic constants

The arrays considered are orthotropic and require 9 elastic constants in 3-D to relate a stress and a strain while the same material in 2-D requires 4 independent constants. These constants are E_x, E_y, ν_{xy} and G_{xy}. The same material with equal orthogonal properties exhibits cubic symmetry and it decreases the constants by 1. Hexagonal arrangement of circular cells in transverse plane introduces isotropy further reducing the constants to 2 viz. E and ν_{xy}.

2.2. Effective flexural rigidity and Differential arc length approximation

The present analysis extends the earlier works of Chung and Waas [5] and Lin and Huang [2] to coated elliptical honeycomb structures. The analysis can be carried on by substituting for an effective flexural rigidity of a section. This is evaluated as a sum of flexural rigidities of the coating and the substrate.The distance of the neutral axis from the lower fiber (inner radius) is

$$\bar{y} = \frac{t_B^2/2 + t_A(t_B/2 + t)(E_A/E_B)}{t}. \quad (1)$$

The effective flexural rigidity, $\widehat{EI}$, is evaluated about the neutral axis and in terms of thicknesses and elastic moduli is [9]

$$\widehat{EI} = \frac{\left(E_A t_A^3 + E_B t_B^3 + 12 E_A t_B \left(t_B - \frac{t\left(\left(\frac{E_A}{E_B}-1\right)t_A + t_B\right)}{2\left(t_A + \frac{E_A}{E_B}t_B\right)}\right)^2 + 3 E_B t_B \left(t_B - \frac{\frac{E_A}{E_B}t_A(t+t_B) + t_B^2}{t_A + \frac{E_A}{E_B}t_B}\right)^2\right)}{12}. \quad (2)$$