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Investigation on buckling behaviors of elastoplastic functionally graded cylindrical shells subjected to torsional loads

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ABSTRACT

In the present work, a semi-analytic solution is presented to analyze buckling behaviors of elastoplastic functionally graded circular cylindrical shells under torsional loads. The material properties vary smoothly through the shell thickness according to the power law distribution and a multi-linear hardening elastoplasticity of materials is included in the analysis. The Ritz energy method and both the flow and deformation constitutive theories help to develop the buckling government equation and the buckling critical condition. An iterative algorithm is resorted to derive the critical load and the buckling mode parameters. Numerical results reveal various effects of the constituent distribution of FGMs, dimensional parameters, and elastoplastic material properties. Meanwhile, the influences of material flow effect on buckling of elastoplastic FGM cylindrical shells are discussed.

1. Introductions

Functionally graded materials (FGMs) are new composites fabricated by mixing ceramic and metallic constituents. The continually varying characteristics of the mixing ratio enable continually grading in their material properties through the shell thickness.

In the recent decades, elastic buckling behaviors of FGM plates and shells had been investigated intensively. Feldman and Aboudi [1] studied buckling behaviors of FGM plates under uniaxial in-plane load. Najafizadeh and Eslami [2] and Najafizadeh and Heydari [3] presented buckling analyses for buckling of circular plates under radial compression. Sofiyev and Schnack [4] investigated dynamic buckling of FGM cylindrical shells under linearly increasing torsional loads. Kadoli and Ganesan [5] considered buckling problems of clamped FGM cylindrical shells under thermal loads. Bagherizadeh et al. [6] investigated buckling issues of FGM cylindrical shells embedded in elastic medium and subjected to combined axial and radial compressive loads. Malekzadeh et al. [7,8] investigated respectively thermal and mechanical load induced buckling behaviors of FGM arbitrary straight-sided quadrilateral plates using differential quadrature method. Uymaz and Aydogdu [9] considered the effects of various boundary conditions on buckling behaviors of three dimensional shear buckling of FG plates. Wu et al. [10] presented linear buckling analysis of simply-supported, multilayered FGM circular hollow cylinders under combined axial compression and external pressure. Yaghoobi and Fereidoon [11] investigated FGM plates resting on elastic foundation for the mechanical and thermal buckling responses. By using Finite element modeling method, Shariyat and Asemi [12] studied shear buckling behaviors of the orthotropic heterogeneous FGM plates resting on the Winkler-type elastic foundation. Meanwhile, some researches focused on the postbuckling issues including geometrical nonlinearity. For instance, Shen [13–16] investigated postbuckling behaviors of FGM cylindrical shells and plates with boundary layer theory considering various mechanical and thermal loads. Woo et al. [17] and Na and Kim [18] focused on postbuckling behaviors of FGM plates induced by thermal loads. Tung [19] presented an analytical approach to investigate the effects of tangential edge constraints on the postbuckling behaviors of FGM flat and cylindrical panels resting on elastic foundations and subjected to thermal, mechanical and thermomechanical loads.

Buckling issue of structures including material nonlinearity is one of the most cumbersome and important components in structural stability theory. Especially in the field of elastoplastic buckling of homogeneous plates and shells, the researches have been very extensive [20–25]. Okada et al. [20] analyzed buckling behaviors of cylindrical vessels under shear forces in the elastic–plastic region. Ma et al. [21] experimentally investigated dynamic plastic buckling of circular cylindrical shells under impact torque. Mao and Lu [22,23] investigated elastoplastic buckling of isotopic cylindrical shells under axial compression and torsional loads with classical shell theory. Currently, literature reporting elastoplastic

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buckling performances of composite plates and shells is limited, especially for FGM ones.

As far as the elastoplastic constitutive model was concerned, Tamura et al. [26] defined the rule of mixtures for metal alloy named TTO model, which was extended to ceramic/metal system by Bocciarelli [27] to describe the elastoplastic behaviors of FGMs. Meanwhile, an inverse analysis procedure based on indentation tests was proposed by Nakamura et al. [28] to identify the constitutive parameters of FGMs. With this model, some literatures were reported concerning thermal stress responses [29–33] and fracture performances of FGMs [34,35].

As FGMs are composed of metallic constituent materials, typically ductile ones, their plastic deformation may be of significant influence on material flow, and therefore, greatly affect buckling behaviors of FGM cylindrical shells. To investigate these effects, buckling analysis of elastoplastic FGM circular cylindrical shells subjected to torsional load is presented.

2. Basic description

For FGM cylindrical shells with length L, mean radius measured from the middle plane R, and thickness h, the coordinate system is set as shown in Fig. 1. The origin is placed on the middle plane of the shell at one of the ends. The coordinate axes x, y, and z are respectively in the axial, circumferential, and the inward normal directions.

The shells are assumed to be simply-supported at both ends, and subjected to a torsional moment M which would arouse an in-plane shear stress. Generally, ceramic constituents are usually brittle materials of relatively higher elastic modulus and strength than those of metallic constituents. As load increasing, the shear stress in the ceramic-rich side would rise faster, and results in plastic flow initiating in this area as shown in Fig. 1.

3. Material constitutive

The constituent distribution of FGMs is usually given according to the power law rule [13]

$$V_c = (0.5 + z/h)^k, \quad V_c + V_m = 1 \tag{1}$$

where k is the power law index, which is a positive real number. It is a critical parameter of constituent distribution. V denotes the volume fraction and their subscripts c, m respectively correspond to the ceramic and metallic constituents.

According to the TTO model, brittle ceramic constituents are assumed to be elastic throughout the deformation and material flow of FGMs is assumed to be primly induced by plastic deformation of ductile metallic constituents. Thus, multi-linear hardening elastoplastic material properties of FGMs [28] can be defined by introducing the ratio of stress to strain transfer parameter q in

$$\begin{aligned} E &= \left(\frac{q+E_c}{q+E_m}E_mV_m + E_cV_c\right)\Big/\left(\frac{q+E_c}{q+E_m}V_m + V_c\right) \\ \nu &= V_m\nu_m + V_c\nu_c \\ \sigma_Y &= \sigma_{Ym}\left(V_m + \frac{q+E_m}{q+E_c}\frac{E_c}{E_m}V_c\right) \\ H &= \left(\frac{q+E_c}{q+H_m}H_mV_m + E_cV_c\right)\Big/\left(\frac{q+E_c}{q+H_m}V_m + V_c\right) \end{aligned} \tag{2}$$

where $E(z)$ is elastic modulus, ν the poison ratio, $\sigma_Y(z)$ yield limit, $H(z)$ the tangent modulus. $q = \tilde{q}E_c$, $\tilde{q}$ is the stress transfer parameter, and $\tilde{q} \geqslant 0$. It should be noted that $\tilde{q} = 0$ represents the FGMs flow plastically once the metallic constituents reach their yield limit.

The most popular elastoplastic constitutive relations of homogeneous materials are J_2 flow theory and J_2 deformation theory. In flow theory, the relation between stresses and strains is defined in the following increment form.

$$\mathrm{d}\varepsilon_{ij} = \frac{1}{2G}\mathrm{d}\sigma_{ij} - \frac{3\nu}{E}\delta_{ij}\mathrm{d}\sigma_m + \frac{3}{4J_2\hat{H}}S_{ij}S_{ld}\mathrm{d}\sigma_{ld} \tag{3}$$

where $\hat{H} = HE/(E-H)$, $G = E/[2(1+\nu)]$. The mean stress $\sigma_m = (\sigma_{xx} + \sigma_{yy} + \sigma_{zz})/3$. S_{ij} is the tensor of stress deviator and the second invariant of stress deviator tensor $J_2 = S_{ij}S_{ij}/2$. δ_{ij} is unit matrix.

In deformation theory, the constitutive relation of FGMs can be given as

$$\varepsilon_{ij} = \frac{3}{2E_s}\sigma_{ij} + \left(\frac{1}{K} - \frac{3}{2E_s}\right)\delta_{ij}\sigma_m \tag{4}$$

in which, the secant modulus in complex stress state $E_s = 3EE_s^0/[3E - (1-2\nu)E_s^0]$, E_s^0 is the secant modulus in the uniaxial tension experiment and the elastic parameter K is defined as $K = E/(1-2\nu)$.

The corresponding incremental form of Eq. (4) reads

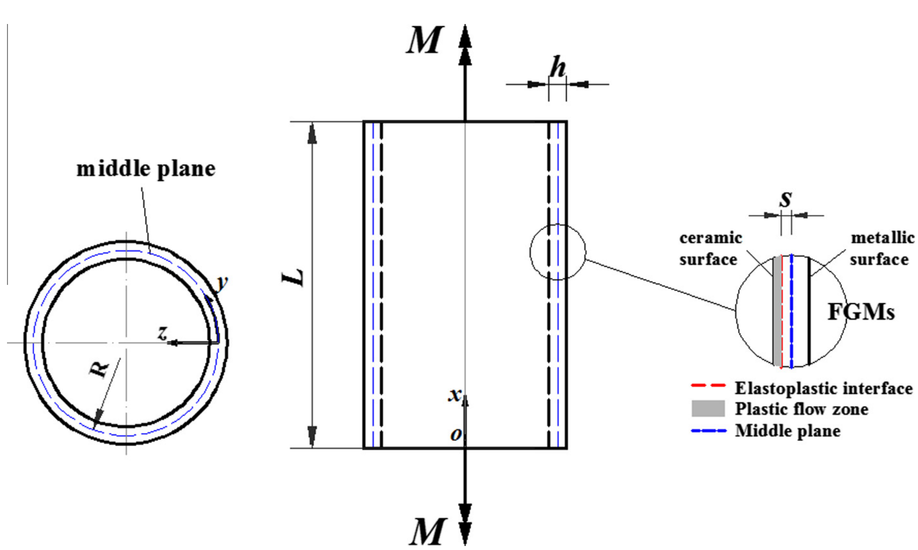

Fig. 1. Geometry of FGM cylindrical shell and the coordinate system.