Composite Structures 118 (2014) 303–315

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

Linear buckling predictions of unstiffened laminated composite cylinders and cones under various loading and boundary conditions using semi-analytical models

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ARTICLE INFO

Article history:
Available online 5 August 2014

Keywords:
Linear buckling
Composite
Cylinders and cones
Axial compression
Torsion
Pressure

ABSTRACT

Semi-analytical models for the linear buckling analysis of unstiffened laminated composite cylinders and cones with flexible boundary conditions are presented. The Classical Laminated Plate Theory and the First-order Shear Deformation Theory are used in conjunction with the Donnell's non-linear equations to derive the buckling equations. Axial, torsion and pressure loads can be applied individually or combined in the proposed models. The stiffness matrices are integrated analytically and for the conical shells an approximation is proposed to overcome non-integrable expressions. Comparisons with the literature show that the classical base functions available for axial compression cannot capture the buckling modes for non-orthotropic laminates. For torsion loads these classical shape functions do not catch the buckling modes even when applying the assumption of pure orthotropy, and it is shown how the proposed models correlate well with experimental data from the literature and finite element results. The use of elastic constraints at the boundaries allows the simulation of different boundary conditions in a versatile way and it is shown how those constants can be adjusted in order to change from one type of boundary condition to another.

1. Introduction

Khdeir et al. [1] were among the first authors to investigate the linear buckling of laminated composite cylindrical shells using higher order equivalent single-layer theories. The authors compare the Classical Laminated Plate Theory (CLPT) with the First-order Shear Deformation Theory (FSDT) and the Third-order Shear Deformation Theory, concluding that using the FSDT already leads to a good prediction of the linear buckling behavior. Geier and Singh [2] are among the first authors to present a linear buckling formulation for composite cylinders using more complete non-linear theories. Using the FSDT the authors compare the Donnell's [3] equations with a deep-shell theory similar to the Sanders' equations [4], achieving analytical expressions for the classical buckling load of simply supported cylinders. In both publications of Khdeir et al. and Geier and Singh, the same set of approximation functions was used to produce the SS2-type of boundary conditions (cf. Table 1) and the orthotropic laminate assumption was applied, where the terms A_{16},A_{26},B_{16},B_{26},$D_{16}$$D_{26}$,$A_{45}$ and the corresponding symmetric counterparts in the laminate constitutive matrix are set to zero. This assumption is required in order to allow the derivation of analytical solutions for the linear buckling problem.

Shadmehri et al. presented in recent studies [5,6] a semi-analytical approach based on the Ritz method for the calculation of the linear buckling load of laminated composite cones under axial compression using the SS3-type of simply supported boundary conditions (cf. Table 1) using the FSDT in conjunction with Donnell's equations, where the cylinder models were verified using the results of Khdeir et al. [1].

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http://dx.doi.org/10.1016/j.compstruct.2014.07.037

Table 1
Elastic constants for the boundary conditions.

SS1: $u = v = w = 0$	$K^u = K^v = K^w = \infty$ $K^{\phi_x} = K^{\phi_\theta} = 0$	CC1: $u = v = w = w_{,x} = 0$	$K^u = K^v = K^w = K^{\phi_x} = \infty$ $K^{\phi_\theta} = 0$
SS2: $v = w = 0$	$K^v = K^w = \infty$ $K^u = K^{\phi_x} = K^{\phi_\theta} = 0$	CC2: $v = w = w_{,x} = 0$	$K^v = K^w = K^{\phi_x} = \infty$ $K^u = K^{\phi_\theta} = 0$
SS3: $u = w = 0$	$K^u = K^w = \infty$ $K^v = K^{\phi_x} = K^{\phi_\theta} = 0$	CC3: $u = w = w_{,x} = 0$	$K^u = K^w = K^{\phi_x} = \infty$ $K^v = K^{\phi_\theta} = 0$
SS4: $w = 0$	$K^w = \infty$ $K^u = K^v = K^{\phi_x} = K^{\phi_\theta} = 0$	CC4: $w = w_{,x} = 0$	$K^w = K^{\phi_x} = \infty$ $K^u = K^v = K^{\phi_\theta} = 0$

The formulation presented in this study is based on the Ritz method and the investigation of different approximation functions is facilitated by the use of a matrix notation. Four models are presented in the framework of CLPT and another four using the FSDT kinematic assumption. For cylindrical shells the new models are compared with the models presented by Khdeir et al., Geier and Singh, whereas for conical shells the comparisons are done with the models of Shadmehri and finite element results. The effect of the orthotropic laminate assumption used by the authors on the obtained eigenvectors is investigated and the limitation of the classical models when this assumption is removed is shown.

As observed by Meyer-Piening et al. [7] the imperfection sensitivity of laminated composite cylinders under axial compression decreases when a combined torsion load is applied, making the linear buckling predictions applicable to imperfection sensitive structures under such combined loads. The authors present test results that were used in the present study to validate the proposed models for combined axial and torsion loads.

Elastic boundary conditions are added to the formulation using one approach which is an extended version of what Som and Deb [8] recently published for isotropic cylinders. A general formulation is presented and constant elastic constraints are assumed along the circumferential direction for all the presented cases. The effect of these elastic constants on the linear buckling behavior using the different proposed models is demonstrated and it is illustrated how the constants can be varied in order to change from one boundary condition type to another, allowing the most flexible models to simulate boundary conditions that require more rigidity.

2. Linear buckling formulation

Fig. 1 shows the coordinate system and the corresponding displacement components, the geometric variables and the applied loads for a general cone, which becomes a cylinder for $\alpha = 0$.

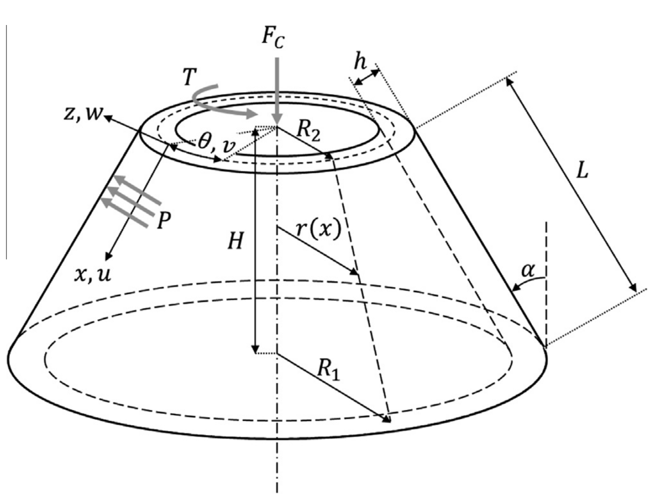

Fig. 1. Cone/cylinder coordinate system and geometric variables.

Using the Ritz method the displacement field is approximated as:

$$\{u\} = [g]\{c\} \tag{1}$$

where $\{u\}^T = \{u \quad v \quad w\}$ for CLPT and $\{u\}^T = \{u \quad v \quad w \quad \phi_x \quad \phi_\theta\}$ for FSDT, $[g]$ contains the base functions used for the approximation and $\{c\}$ the corresponding amplitudes. Since $[g]$ is known the solution involves finding $\{c\}$, i.e. the Ritz constants [9].

The strain field $\{\varepsilon\}$ can be directly calculated from the Ritz constants [10]:

$$\{\varepsilon\} = \left([B_0] + \frac{1}{2}[B_L]\right)\{c\} \tag{2}$$

where $[B_0]$ (defined in Appendix A) and $[B_L]$ are matrices containing the linear and non-linear kinematic terms, respectively. It can be shown that [10]:

$$\begin{aligned}\{\delta\varepsilon\} &= [\overline{B}]\{\delta c\} \\ [\overline{B}] &= [B_0] + [B_L]\end{aligned} \tag{3}$$

The non-linear strain matrix $[B_L]$ can be written as the product of a term depending on the Ritz constants $[A]$ and a constant term $[G]$ as:

$$[B_L] = [A][G] \tag{4}$$

such that:

$$[\delta\overline{B}] = [\delta B_L] = [\delta A][G] = [\delta A][G_d][g] \tag{5}$$

with $[A]$ and $[G_d]$ defined in Appendix A for CLPT and FSDT, using Donnell's and Sanders' equations.

The total potential energy of a conservative system (Π) can be represented as the sum of the strain energy and the work of the applied loads V (external) as:

$$\Pi = U + V \tag{6}$$

The linear buckling behavior can be calculated applying the neutral equilibrium criterion[11,6]:

$$\delta^2\Pi = 0 \tag{7}$$

to the total potential energy equation, giving the expression:

$$\delta(\delta U + \delta V) = 0 \tag{8}$$

The first variation of the strain energy (δU) and the variation of the eternal energy δV, for the system illustrated in Fig. 1, can be expressed as:

$$\begin{aligned}\delta U &= \int\int_{x\theta} \{\delta\varepsilon\}^T\{N\} r d\theta dx = \{\delta c\}^T \int\int_{x\theta} [\overline{B}]^T\{N\} r d\theta dx \\ \delta V &= \{\delta c\}^T\{f_{ext}\}\end{aligned} \tag{9}$$

where $\{N\}$ is the vector of the distributed forces and moments and $\{\varepsilon\}$ the vector of the corresponding strains and rotations, both detailed in Appendix A. Substituting Eq. (9) into Eq. (8) and calculating the second variations gives:

$$\{\delta c\}^T\left(\int\int_{x\theta} [\overline{B}]^T\{\delta N\} r d\theta dx + \int\int_{x\theta} [\delta\overline{B}]^T\{N\} r d\theta dx\right) = 0 \tag{10}$$