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# Stress concentration phenomena in the vicinity of composite plate-doubler junctions by a layerwise analysis approach

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ABSTRACT

This paper presents an analysis method for the determination of displacements, strains and stresses in a laminated composite plate subjected to tensile load that is reinforced by a doubler. The analysis approach consists of two parts. Firstly, a 'global' solution that is based on Classical Laminated Plate Theory (CLPT) is introduced. Secondly, a 'local' model is derived that enables the assessment of the three-dimensional stress state near the plate-doubler junction. The local solution employs a discretization of the physical layers into a number of mathematical layers which necessitates the numerical treatment of a quadratic eigenvalue problem. Consequently, the current approach can be classified as being a semi-analytical layerwise analysis method. Results are generated for several different plate-doubler configurations, and it is found that the current analysis model delivers excellent results when compared to finite element simulations, however with only a fraction of the computational time and effort that are needed for the FEM analyses.



## 1. Introduction

Composite materials in the form of fiber-reinforced plastics (FRP) – i.e. a matrix material (for instance epoxy) which is being reinforced by unidirectional fibers (e.g. carbon fibers) – have found an increasing use in many engineering branches due to their superior properties in terms of high strength-to-weight and stiffness-to-weight ratios. Naturally, such FRP which often appear in the form of laminated composite plates or shells (i.e. thin-walled structures that consist of a number of FRP-layers) are very attractive for such applications where the structural weight plays an important role. Of course, this holds true especially in lightweight engineering branches such as aeronautics and astronautics, but also in automotive engineering or in the wind energy sector.

In many cases it is necessary to reinforce a laminated plate or shell at certain locations in order to ensure their structural integrity. As an example, such reinforcements may be the result of a structural repair at locations where a laminate has been damaged and where after the repair measures have been completed a patch/doubler is being attached in order to additionally strengthen the plate or shell. In certain cases reinforcements may also be necessary due to static requirements whenever a stronger cross-section is required locally so that the applied loads can be carried by the structure without failure, or when a hole through the thickness of the laminate requires reinforcements. Two general overview papers concerning repair concepts for composite structures are available with Myhre and Beck [20] and Myhre and Labor [21].

In Fig. 1 a typical structural situation consisting of a laminated plate ('basic structure') and a rectangular doubler (i.e. an external repair solution) is depicted, where it should be noted that the planform of doublers in composites engineering can take up virtually any form (i.e. circular, quadratic, rectangular, elliptic, polygonial), depending on the application case and the specific requirements. Further, a distinction has to be made between tapered repair patches (i.e. such patches where there is no abrupt change of thickness at the plate-doubler junction but rather a smooth transition from the reinforced structure to the basic structure is ensured, see for instance He et al. [7]) and such patches where no tapering is used and where the change in thickness at the plate-doubler junction is discontinuous and abrupt. The present contribution is concerned with the stress analysis of the latter type of patch reinforcements.

Naturally, the presence of the doubler will lead to significant stress concentrations in the vicinity of the plate-doubler junction due to the geometrical discontinuity and the abrupt change in the thickness of the structure which needs to be taken into account carefully whenever such patches/doublers are applied. These stress

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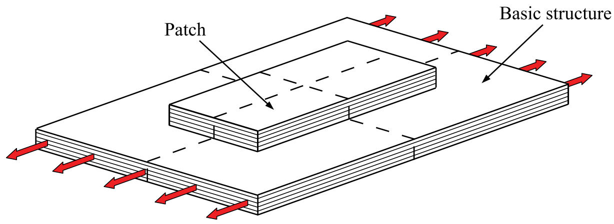


**Fig. 1.** An example for a laminated plate reinforced by a rectangular doubler.

concentrations will manifest themselves in the form of significant three-dimensional stress states at the edges and corners of the interface between plate and doubler, even though the far-field loading might consist of a simple load case such as uniaxial tensile load as exemplarily shown in Fig. 1. Further, due to the presence of the free edges of the doubler, it can be expected that free-edge stress fields will arise in this area as well which must be taken into consideration when designing and analyzing reinforcements in the form of doublers.

Free-edge effects are mainly triggered by the inherent anisotropy and thus the general mismatch of the elastic properties of adjacent layers in a composite laminate and are characterized by very localized and potentially singular interlaminar stress fields in the vicinity of the traction free edges of composite laminates. The free-edge effect has been under investigation by the scientific community since the pioneering publication by Pipes and Pagano [24] in which a plane laminated specimen under uniaxial tension has been treated by a finite difference formulation, and a virtually uncountable number of papers employing a broad variety of closed-form analytical, semi-analytical, purely numerical, but also experimental approaches is available today. An encompassing literature survey is well beyond the scope of this paper, and a good number of survey papers on this specific topic is available a selection of which is cited with Salamon [29], Pagano [22], Herakovich [9], Reddy and Robbins [25], Kant and Swaminathan [12], Mittelstedt and Becker [16], Mittelstedt and Becker [18]. The interested reader is advised to consult these references in order to gain a deeper insight into the state of the art concerning the modeling and analysis of free-edge effects in composite laminates.

The closed-form analytical, semi-analytical and numerical modeling as well as the experimental study of composite structures that are reinforced by doublers has been the topic of a good number of publications in the last decades. In the following a short selective overview of relevant references is given where it must be noted that no claim of completeness is made here.

A rather simple and straightforward approach for assessing the strength of a repair solution of a thin-walled composite laminated structure based on a stress and strain calculation in the framework of Classical Laminated Plate Theory (short: CLPT) in conjunction with stress-based failure criteria was reported by Robson et al. in [28]. Soutis and Hu [30] presented a closed-form analytical method for the determination of interlaminar stresses in plate-doubler interfaces based on a rather simple shear-lag analysis approach. Further, they employed a refined three-dimensional finite element mesh for the determination of stress concentration factors and critical locations at round external patch repairs and studied the influence of the thickness of the repair patch on the stress fields. In a follow-up paper, Soutis et al. [31] performed experiments on composite laminates under compressive load that were repaired by circular patches. Furthermore, by using a full-scale three-dimensional finite element model Soutis et al., performed a stress analysis of the repaired region in order to predict the location, type and initiation of damage in the vicinity of the plate-doubler interface. Hu and Soutis [10] again investigated the compressive behavior of laminated composite specimens under compressive load that were strengthened by a circular patch and used a shear lag analysis approach in order to derive design guidelines for an optimized choice of patch geometry, size and inplane stiffness. A three-dimensional finite element model in conjunction with a cohesive zone approach delivered results for the stress fields in the vicinity of the plate-doubler junction and enabled an estimate of the ultimate compressive loads of such repaired specimens. Lee and Knauss [13] performed experimental and numerical three-point and four-point bending studies on composite laminated specimens which included abrupt thickness changes and determined the load level at which damage initiation takes place for combined loading conditions including tension, bending and interlaminar shear. Tsamasphyros et al., [34] performed elasticity-based analytical calculations and numerical computations of isotropic plates that contain cracks and that are reinforced by composite patches, wherein patches on one side of the plate as well as patches attached to both sides were considered. A closed-form analysis of the inplane stress fields in the vicinity of patch repairs of composite laminated plates was presented by Engels and Becker [6] who investigated composite containing an elliptical hole which is reinforced by an elliptical doubler and used the complex potential method for deriving a closed-form analytical analysis method. However, their analysis was restricted to the pure inplane stress state, and interlaminar stresses were not part of the investigations. A similar analysis approach allowing for arbitrarily shaped holes and patches was presented by Zemlyanova [38]. Duong and Yu [5] investigated the thermal stresses that arise in a composite bonded repair where an octagonal patch is applied on a cracked plate by means of an elasticity based approach. Again, the analysis approach only captured inplane stress fields, the typical interlaminar stresses at plate-doubler junctions were not part of the investigations. A numerical model for the assessment of the stress fields in the vicinity of cracked plates that are being repaired and reinforced by composite patches was developed by Bachir Bouiadjra et al. [2]. Specifically, the cracks were considered under pure mode I as well as under mixed mode loading, and the stress intensity factors occurring in this region were determined. Mathias et al. [15] reported the application of genetic algorithms to the optimization of composite patches applied to aluminum plates containing circular holes. The aim of the optimizations was to reduce the stresses in the vicinity of the holes considering constraints in terms of the patch shape, size and location by altering the stacking sequence and size of the patch. In the same way Brighenti [3] optimized patches attached to cracked plates under mode I and mode II loading and determined optimum topologies in order to maximize the fracture resistance and fatigue life behavior. Wigger and Becker [36,37] performed asymptotic investigations of the inplane stress singularities in the vicinity of corners of doublers attached to composite plates and used the complex variables method and the boundary finite element method, respectively. They showed that even though the orders of the occurring stress singularities are generally far lower than for typical singular stress problems in the vicinity of crack tips, they may become significant nonetheless, and the corners of the plate-doubler junctions are generally prone to delamination failure. Note that in these investigations the complete three-dimensional stress field and the according stress singularities were not considered. Papanikos et al. [23] developed a numerical model for the progressive failure analysis of isotropic plates containing through-the-thickness cracks that were repaired by tapered composite patches attached to both sides of the plates. A study from a different application field and yet related to the scope of the present paper was published by Carpinteri et al. [4] who investigated the structural behavior of beams with CFRP reinforcements. Specifically, Carpinteri et al., considered the debonding behavior of the CFRP reinforcements due to the stress concentrations at the interface