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Four-node shell element for doubly curved multilayered composites based on the Refined Zigzag Theory

Daniele Versino [a,*], Marco Gherlone [b], Marco Di Sciuva [b]

[a] Theoretical Division, T-3, Los Alamos National Laboratory, MS B216, Los Alamos, NM 87545, USA
[b] Department of Mechanical and Aerospace Engineering, Politecnico di Torino, Corso Duca degli Abruzzi 24, 10129 Torino, Italy

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ABSTRACT

In the present paper a generalization of the Refined Zigzag Theory (RZT) to doubly-curved multilayered structures is proposed. The displacement field characteristic of Naghdi's shell model is enriched with RZT kinematics and a four-node shell finite element is formulated. Assumed Natural Strain (ANS) strategy is employed to overcome shear locking and Enhanced Assumed Strain (EAS) technique is applied to alleviate membrane locking and bending locking. For efficiency purpose, a one-point quadrature rule is used for the in-plane integration and hourglass stabilization is introduced. Finally, several numerical examples, involving static analysis of thick as well as thin shells, are performed to demonstrate the efficiency and accuracy of the proposed shell finite element.

1. Introduction

In the last decades there has been a recognizable increase in the application of composite materials within a lot of engineering fields (aerospace and marine, civil and defense). This success is mainly due to the fact that composite materials are stiff and resistant light-weight materials and that they can be easily tailored in order to be optimized for any specific use. The typical structural typology built up with composite materials is based on a thickness-wise sequence of layers with different mechanical properties (transverse anisotropy). Taking into account that composite materials also exhibit a transverse deformability that is higher than for metallic materials, the modeling effort is particularly challenging.

The usual classifications adopted for the one- and two-dimensional models of multilayered composite structures are based on the nature of the primary unknowns and on the assumptions made on these unknowns. There are displacement-based theories (only the displacements are primary unknowns) [1] and mixed theories (both the displacements and the transverse stresses may be unknowns) [2]. The focus of the present work will be on displacement-based theories. Moreover, depending on the through-the-thickness description of the displacement and stress fields, there may be: Equivalent Single Layer theories (ESL, a through-the-thickness distribution of the unknowns is assumed over the whole laminate thickness) and Layer-Wise theories (LW, the distribution of the unknowns is assumed layer by layer) [3].

The Classical Lamination Plate Theory (CLPT) [4] and the First-order Shear Deformation Theory (FSDT) [5,6] are easy-to-implement displacement-based ESL theories widely adopted for research and engineering applications. Moreover, most of the finite elements present into commercial codes are based on the FSDT. Nevertheless, both CLPT and FSDT do not model accurately transverse strains and stresses: CLPT totally neglects transverse deformability whereas FSDT produces piecewise constant transverse shear stresses, thus violating the requirement of transverse stresses through-the-thickness continuity. For these reasons, CLPT and FSDT may be used for thin and moderately thick laminated structures with reduced transverse anisotropy and their applicability is often limited to global responses (maximum deflection, vibration frequencies, buckling loads). Higher-order through-the-thickness expansions of the displacements (polynomial [7,8] or trigonometric [9]) may lead to improved predictions but transverse stresses continuity is not satisfied (unless integration of indefinite equilibrium equations is applied to recover accurate transverse stresses).

LW theories based on displacements [10,11] can guarantee a high degree of accuracy, but at the expense of computational complexity. The number of unknowns in LW theories increases with the number of layers thus becoming prohibitive for complex analyses (non-linear, progressive failure) on laminated structures with several layers.

An interesting compromise between high accuracy and low computational costs is represented by the so-called zigzag

* Corresponding author. Tel.: +1 505 667 2137; fax: +1 505 665 5926.
E-mail address: daniele.versino@lanl.gov (D. Versino).

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theories. These theories were pioneered in the early eighties by Di Sciuva [12–14]: transverse normal deformability is usually neglected and transverse displacement is through-the-thickness constant whereas in-plane displacements are the result of the superposition of a through-the-thickness polynomial distribution (linear [13] or cubic [14]) and a piecewise linear and continuous (i.e., zigzag) contribution, introduced in order to model the thickness-normal distortion typical of multilayered structures. The zigzag contribution to the in-plane displacement is shaped in order to guarantee that transverse shear stresses are through-the-thickness continuous. The total number of unknowns does not depend on the number of layers. Inspired by Di Sciuva's works, a great number of zigzag theories have been proposed over the last two decades [15–23].

Recently, a further development has been proposed within the family of zigzag models, namely the Refined Zigzag Theory (RZT) [24–27], in order to consistently address some drawbacks of the previously presented approaches. In fact, the transverse shear stresses obtained from the constitutive equations of classical zigzag theories vanish erroneously at clamped boundaries and this has been ignored by many authors (apart from [16,17]). Within RZT, transverse shear stresses are allowed to be piecewise constant and discontinuous at the layer interfaces. Nevertheless, more accurate predictions of all response quantities are obtained, including the transverse shear stresses, also at clamped boundaries [24,26,28]. Moreover, most zigzag theories require C1-continuous finite element approximations, the type of approximation that is particularly undesirable for plate and shell elements. Some of the already mentioned works have faced this problem [16,17,23]. A number of finite element formulations based on RZT have been presented [29–34], all of them requiring only C0-continuity of shape functions and thus being suitable for inclusion into FEM commercial codes [31]. Finally, a great versatility of RZT has been demonstrated, since the zigzag contribution to the in-plane displacements can be naturally and accurately adapted to some special cases, including homogeneous laminates [27] and laminates with external shear-deformable layers [35]. Further enhancements of RZT can be found in [36,37].

In the present paper a generalization of RZT to doubly-curved multilayered shells is introduced. A four-node shell element based on the proposed formulation is developed employing bilinear shape functions for the in-plane interpolation of the seven kinematic variables. Therefore, seven degrees of freedom are defined at each node of the element, namely three displacements, two average rotations and two zig-zag amplitudes. The current formulation may be regarded as an extension of previous work by Versino et al. [31]. Shear locking due to the bilinear interpolation is mitigated using the Assumed Natural Strain (ANS) interpolation [32,38,39]. Moreover, the element's performances in membrane- and bending-dominated problems are improved applying the Enhanced Assumed Strain (EAS) methodology [40,41]. In-plane integration is carried out with a one-point quadrature rule and hourglass stabilization is employed [42,43].

The paper is organized as follows. In the next section, the problem is described and the notation is defined. In Section 3, geometric and kinematic quantities of the shell are introduced through a tensorial representation. The displacement field of the RZT-based shell is presented in Section 4. Subsequently, the variational formulation of the equilibrium equations is derived in Section 5, and the resulting computational formulation is derived in Section 6. The performances of the present formulation are assessed by way of several numerical experiments, involving thick as well as thin shells, in Section 7. Finally, concluding remarks are presented in Section 8.

2. Notation

A curved laminated shell of thickness $2h$ is composed of N perfectly-bonded layers. The N layers are separated by $N-1$ interfaces and the notation $(\bullet)^{(k)}$ is used to indicate a quantity associated with the k-th layer or interface. The Cartesian coordinates of every point belonging to the shell's reference surface, Ω, are given by its position vector $\boldsymbol{r}(\xi^1,\xi^2)$, where ξ^1, ξ^2 are curvilinear coordinates defined on the closed domain Ω. The ξ^3-coordinate spans the range $[-h,h]$, and is directed along the unit vector $\boldsymbol{a}_3(\xi^1,\xi^2)$, normal to the reference surface (see Fig. 1). The thickness of the kth layer is $2h^{(k)} = \xi^{3(k)} - \xi^{3(k-1)}$, the kth interface is situated between layers k and $k+1$ and the top and bottom surfaces of the shell are respectively located at $\xi^{3(N)} = h$ and $\xi^{3(0)} = -h$. The Cartesian coordinates of every point in the closed domain occupied by the shell, $\mathscr{B} = \Omega \times [-h,h]$, are hence given by its position vector,

$$\boldsymbol{R}(\xi^1,\xi^2,\xi^3) = \boldsymbol{r}(\xi^1,\xi^2) + \xi^3\boldsymbol{a}_3. \tag{1}$$

The domain's boundary, $\partial\mathscr{B}$, is split into two disjoint subsets, $\partial\mathscr{B}_u = \partial\Omega_u \times [-h,h]$ and $\partial\mathscr{B}_\sigma = \partial\Omega_\sigma \times [-h,h]$, where essential, $\boldsymbol{u} = \bar{\boldsymbol{u}}$, and natural boundary conditions, $\boldsymbol{\sigma}\cdot\boldsymbol{a}_3 = \bar{\boldsymbol{h}}$, respectively, are enforced. Finally, it is noted that in the following, unless otherwise stated, Greek indices vary from 1 to 2, while their Latin counterparts vary from 1 to 3.

3. Kinematic description

It is recalled from the previous section that the midsurface of the shell is defined by the mapping, $\boldsymbol{r}$, of *convected* curvilinear coordinates, ξ^α, into Cartesian coordinates in $\mathbb{R}^3$. The covariant base vectors of the tangential plane are thus given by [44]:

$$\boldsymbol{a}_\alpha = \frac{\partial \boldsymbol{r}}{\partial \xi^\alpha}. \tag{2}$$

The components of the surface metric tensor (first fundamental form) are defined as

$$a_{\alpha\beta} = \boldsymbol{a}_\alpha \cdot \boldsymbol{a}_\beta \tag{3}$$

and the following properties hold:

$$a^{\alpha\beta} = \left(a_{\alpha\beta}\right)^{-1}, \quad \boldsymbol{a}^\alpha = a^{\alpha\beta}\boldsymbol{a}_\beta, \quad \boldsymbol{a}_\alpha \cdot \boldsymbol{a}^\beta = \delta^\beta_\alpha, \tag{4}$$

where δ^β_α is Kronecker's delta tensor. Moreover, the unit normal vector is given by

$$\boldsymbol{a}_3 = \frac{\boldsymbol{a}_1 \times \boldsymbol{a}_2}{\|\boldsymbol{a}_1 \times \boldsymbol{a}_2\|} \tag{5}$$

and an element of surface area is obtained as

$$\mathrm{d}\Omega = \|\boldsymbol{a}_1 \times \boldsymbol{a}_2\| \mathrm{d}\xi^1 \mathrm{d}\xi^2 = \sqrt{\det(a_{\alpha\beta})}\, \mathrm{d}\xi^1 \mathrm{d}\xi^2. \tag{6}$$

In light of the mapping (1), three-dimensional covariant base vectors for the ξ-reference frame are given by

$$\boldsymbol{g}_i = \frac{\partial \boldsymbol{R}}{\partial \xi^i}, \tag{7}$$

the components of the metric tensor are given in covariant form as

$$g_{ij} = \boldsymbol{g}_i \cdot \boldsymbol{g}_j \tag{8}$$

and, similarly to Eq. (4), the following properties hold:

$$g^{ij} = \left(g_{ij}\right)^{-1}, \quad \boldsymbol{g}^i = g^{ij}\boldsymbol{g}_j, \quad \boldsymbol{g}_i \cdot \boldsymbol{g}^j = \delta^j_i. \tag{9}$$

Substituting Eq. (1) into (7), the covariant base vectors $\boldsymbol{g}_i$ may be expressed as follows