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Implementation of the refined zigzag theory in shell elements with large displacements and rotations

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ABSTRACT

This work shows a possible implementation of the refined zigzag theory in elements based on Simo's shell theory. Refined zigzag theory can deal with composite laminate economically, adding only two nodal degrees of freedom, with very good accuracy. Two existing elements are considered, a four-node bi-linear quadrilateral and a six-node linear triangle. This geometry is enhanced with a hierarchical field of in-plane displacement expressed in convective coordinates. The objective is to have simple and efficient elements to analyze composite laminates under large displacements and rotations but small elastic strains. General aspects of the implementation are presented, and in particular the assumed natural strain technique used to prevent transverse shear locking. Several examples are considered to compare on the one hand with analytical static solutions and natural frequencies of plates, and on the other hand to observe the buckling loads and non-linear behavior with large displacement in double curved shells. In these latter cases comparisons are against numerical solutions obtained with solid elements. The results obtained are in a very good agreement with the targets used.

1. Introduction

Both the classical theory of thin shells [7] and the first order transverse shear deformation theory (FSDT, [12]) lead to good results in treating homogeneous materials. However, the basic hypothesis that assumes that fibers in the direction normal to the shell will remain straight leads to poor predictions when dealing with materials with a high degree of heterogeneity across the thickness.

In order to improve predictions, theories with higher order (cubic or higher) interpolation of displacements in the thickness of the shell have been proposed (see for example the monograph [10]) but their use is not widespread because their predictive power is unreliable.

A three-dimensional analysis using solid elements appears as the most suitable technique for the treatment of composite materials, but it can easily become prohibitively expensive due to the number of layers in the laminate that may be greater than 100. In such cases multiple layers may be grouped together within one single layer with combined properties in order to maintain the number of degrees of freedom (DoFs) of the problem within manageable limits [8].

More precise techniques than those based on shell theories are layer-wise approaches, in which the thickness of the laminate is divided into a number of layers (which may or may not coincide with the physical number of layers) assuming a linear variation of displacements (in the plane of the layer) between layers. A review of these techniques can be seen in [11]. This approach clearly suffers from the same problem of using three-dimensional solid elements for the analysis.

The analysis made with solid element models and layer-wise approaches show that the profile of the in-plane displacement along the normal to the plane of the laminate can not be approximated by a polynomial of higher order. That has led to the appearance of zigzag approximations where the interpolation functions are only C^0 continuous across the thickness and with a zigzag profile, i.e. the first derivative (with associated transverse shear deformation) is discontinuous. This naturally occurs due to the different modules of transverse elasticity of each layer, which may differ by several orders of magnitude. In [1] a review of these theories may be seen. Recently a refined version of this proposal [15] has been presented, that based on the FSDT (5 DoFs), two additional DoFs are included, representing the amplitudes of hierarchical in-plane

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displacements added to the linear through-the-thickness assumption kinematics. This theory involves constant transverse shear stresses in each layer (and therefore discontinuous) but allows treating clamped boundary conditions that is a limitation present in early zigzag theories.

These refined zigzag theories (ZZRT) have been implemented in 2D beam elements [5,9] and flat plate elements [15,2,6] where a very good approximation has been reported for the displacement field across the laminate thickness. It was also reported that the shear stresses directly obtained using the constitutive relation and the shear deformations computed from the displacement field show a poor approximation. An accurate recovery of transverse shear stresses requires integration across the laminate thickness of the equilibrium equations in the plane of the sheet, for which the derivatives of the stresses between finite elements must be evaluated.

In the author's knowledge the ZZRT has not been used in double curvature shells. Furthermore geometrically nonlinear models for plates resort to the use of von Kármán plate kinematics in order to evaluate buckling loads.

In this paper a possible implementation of the ZZRT on shell elements based on the geometrically exact shell theory of Simo [13] is presented. The elements considered are a bi-linear 4-node quadrilateral and a linear 6-node triangle, the latter with a no-conforming interpolation of the field director [4]. The scope of this work is restricted to small elastic strains but large displacements and rotations.

An outline of this paper is as follows. The basic kinematic of the base shell theory by Simo (FSDT) is summarized in the next section. Then additional displacement fields (ZZRT) are introduced and a possible way to obtain the across the thickness interpolation is explained. Resulting elasticity matrices for the new generalized stress and strain measures are then evaluated. Section 6 summarizes the base shell elements used and the modifications required to include the ZZRT approximation while in Section 7 the transverse shear approach to avoid locking is explained. Several examples are presented in Section 8 to compare linear plate models with theoretical results and non-linear shell models with 3D solid discretizations. Finally some conclusions are summarized.

2. Basic kinematics

For a formulation in large displacements and rotations and small strains compatible with an elastic composite laminate, we start from the approach proposed by Simo et al. [13] where the configuration of the shell is defined by the position of the middle surface $\boldsymbol{\varphi}$ and the field director $\mathbf{t}$ (pseudo normal). The positions of a material point before and after the deformation are written as

$$\mathbf{X}(x,y,z) = \boldsymbol{\varphi}_0(x,y) + z\mathbf{t}_0(x,y) \tag{1}$$

$$\mathbf{x}(x,y,z) = \boldsymbol{\varphi}(x,y) + z\mathbf{t}(x,y) \tag{2}$$

$$\boldsymbol{\Lambda}(x,y) = [\mathbf{t}_1 \quad \mathbf{t}_2 \quad \mathbf{t}_3] \tag{3}$$

where (x,y,z) are convective coordinates in a suitably chosen local system with z in the direction of the director and (x,y) are associated with two orthogonal directions in the tangent plane to the middle surface. The director $\mathbf{t}$ is the third component ($\mathbf{t}_3$) of the local triad $\boldsymbol{\Lambda}$ and it is assumed that for small strains the thickness does not change during the deformation.

The relevant strain measures are obtained first evaluating the deformation gradient relative to the convective system

$$\mathbf{F} = \frac{\partial \mathbf{x}}{\partial \mathbf{X}} = \left[\boldsymbol{\varphi}_x + z\mathbf{t}_x, \boldsymbol{\varphi}_y + z\mathbf{t}_y, \mathbf{t}\right] \tag{4}$$

and with it the right Cauchy–Green tensor results

$$\mathbf{C} = \mathbf{F}^T\mathbf{F} = \begin{bmatrix} \boldsymbol{\varphi}_x\cdot\boldsymbol{\varphi}_x & \boldsymbol{\varphi}_x\cdot\boldsymbol{\varphi}_y & \boldsymbol{\varphi}_x\cdot\mathbf{t} \\ \boldsymbol{\varphi}_y\cdot\boldsymbol{\varphi}_x & \boldsymbol{\varphi}_y\cdot\boldsymbol{\varphi}_y & \boldsymbol{\varphi}_y\cdot\mathbf{t} \\ \mathbf{t}\cdot\boldsymbol{\varphi}_x & \mathbf{t}\cdot\boldsymbol{\varphi}_y & 1 \end{bmatrix} + z\begin{bmatrix} \mathbf{t}_x\cdot\boldsymbol{\varphi}_x & \mathbf{t}_x\cdot\boldsymbol{\varphi}_y & 0 \\ \mathbf{t}_y\cdot\boldsymbol{\varphi}_x & \mathbf{t}_y\cdot\boldsymbol{\varphi}_y & 0 \\ 0 & 0 & 0 \end{bmatrix} + z^2\begin{bmatrix} \mathbf{t}_x\cdot\mathbf{t}_x & \mathbf{t}_x\cdot\mathbf{t}_y & 0 \\ \mathbf{t}_y\cdot\mathbf{t}_x & \mathbf{t}_y\cdot\mathbf{t}_y & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{5}$$

where the terms associated with z^2 are commonly neglected. Then it can be distinguished

- The metric tensor of the middle surface

$$\begin{bmatrix} a_{xx} \\ a_{yy} \\ 2a_{xy} \end{bmatrix} = \begin{bmatrix} \boldsymbol{\varphi}_x\cdot\boldsymbol{\varphi}_x \\ \boldsymbol{\varphi}_y\cdot\boldsymbol{\varphi}_y \\ 2\boldsymbol{\varphi}_x\cdot\boldsymbol{\varphi}_y \end{bmatrix} \tag{6}$$

and the Green Lagrange strain tensor of the middle surface

$$\begin{bmatrix} E_{xx} \\ E_{yy} \\ 2E_{xy} \end{bmatrix} = \frac{1}{2}\begin{bmatrix} a_{xx}-1 \\ a_{yy}-1 \\ 2a_{xy} \end{bmatrix} = \mathbf{E}_m \tag{7}$$

- the pseudo curvature tensor

$$\begin{bmatrix} \kappa_{xx} \\ \kappa_{yy} \\ 2\kappa_{xy} \end{bmatrix} = \frac{1}{2}\begin{bmatrix} \mathbf{t}_x\cdot\boldsymbol{\varphi}_x \\ \mathbf{t}_y\cdot\boldsymbol{\varphi}_y \\ \mathbf{t}_x\cdot\boldsymbol{\varphi}_y + \mathbf{t}_y\cdot\boldsymbol{\varphi}_x \end{bmatrix} \tag{8}$$

that allows to compute curvature changes from the original configuration

$$\begin{bmatrix} \chi_{xx} \\ \chi_{yy} \\ 2\chi_{xy} \end{bmatrix} = \begin{bmatrix} \kappa_{xx} \\ \kappa_{yy} \\ 2\kappa_{xy} \end{bmatrix} - \begin{bmatrix} \kappa^0_{xx} \\ \kappa^0_{yy} \\ 2\kappa^0_{xy} \end{bmatrix} = \boldsymbol{\chi} \tag{9}$$

- the transverse shear strains

$$\begin{bmatrix} a_{zx} \\ a_{zy} \end{bmatrix} = \begin{bmatrix} \mathbf{t}\cdot\boldsymbol{\varphi}_x \\ \mathbf{t}\cdot\boldsymbol{\varphi}_y \end{bmatrix} \tag{10}$$

$$\begin{bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{bmatrix} = \begin{bmatrix} \mathbf{t}\cdot\boldsymbol{\varphi}_x - \mathbf{t}^0\cdot\boldsymbol{\varphi}^0_x \\ \mathbf{t}\cdot\boldsymbol{\varphi}_y - \mathbf{t}^0\cdot\boldsymbol{\varphi}^0_y \end{bmatrix} = \begin{bmatrix} a_{zx} \\ a_{zy} \end{bmatrix} - \begin{bmatrix} a^0_{zx} \\ a^0_{zy} \end{bmatrix} \tag{11}$$

These generalized strains allow to obtain the strain tensor at any point across the thickness.

3. Additional displacement field

To consider the use of the ZZRT it is necessary to distinguish displacements in the direction of the director and displacements in the tangent plane to the middle surface. Besides, these new displacements are additional to the basic kinematics (i.e. hierarchical DoFs). The zigzag functions are introduced into the convective local coordinate system with components in the tangent plane of the shell (directions (x,y))

$$\begin{bmatrix} u(x,y,z) \\ v(x,y,z) \end{bmatrix} = \begin{bmatrix} \phi_x(z) & \\ & \phi_y(z) \end{bmatrix}\begin{bmatrix} \psi_x(x,y) \\ \psi_y(x,y) \end{bmatrix} \qquad \mathbf{u}(x,y,z) = \boldsymbol{\phi}(z)\boldsymbol{\psi}(x,y) \tag{12}$$

where $\boldsymbol{\psi}$ is the amplitude of the hierarchical displacement and the hierarchical interpolation function across the thickness $\phi_i(z)$ (zigzag function) is null at both bottom and top shell surfaces

$$\boldsymbol{\phi}\left(\pm\frac{h}{2}\right) = \mathbf{0} \tag{13}$$