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A specific finite element procedure for the analysis of elastic behaviour of short fibre reinforced composites. The Projected Fibre approach

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ABSTRACT

Motivated by computing elastic properties of reinforced natural fibre composites, a multi-scale numerical model, named 'the Projected Fibre approach (PF)', is proposed. It uses a specific finite element procedure which is associated with a random distribution of short fibres. It takes into account of the geometry and the mechanical properties of composite's components. A microscopic truss finite element is used to model the short fibre reinforcements. The corresponding degrees of freedom are projected on those of the resin matrix. Numerical results of the elastic properties of a reinforced hemp fibre polypropylene composite are compared to those obtained from the experiment.

1. Introduction

Natural fibres as reinforcement for composites with thermoplastic matrices are currently exploited in place of glass and/or other synthetic materials, particularly in non-structural applications. Several application sectors become more and more interested by a possible future use of bio-composites materials, especially those reinforced with natural fibres (woven composites, randomly distributed fibre composites…). Among them, sectors of automotive, construction, goods, biomedical and packaging seem to be the most cited on these applications [1–6]. The main advantage of employing natural fibres is that these are biodegradable and renewable, and exhibit low cost, low density, high toughness and good thermal resistance. Moreover polymer materials reinforced with natural fibres (hemp, flax, sisal, wood-fibre, yute, alfa, miscanthus…) can combine satisfactory mechanical properties with a low specific mass. So far studies on the properties of natural fibres based composites have been the subject of a large number of papers and reviews, especially during the last decade [7]. Prediction of macroscopic properties of composite materials (Young's modulus, Poisson's ratio, shear modulus,…) from those of components is one of the main objectives of modelling. The mechanical behaviour of such materials under loading derives from active mechanisms inside their components and at the interfaces, as well as from the arrangement of these components. The prediction of macroscopic behaviour from these data uses complex operations of scale change which represent the interaction phenomenon between components. The micromechanical approaches are the first ones to be proposed in the literature, for estimating elastic properties of short fibre reinforced composites. We mention some models known from the literature which are based, for most of them, on two basic assumptions: the matrix and fibre are linearly elastic and a random distribution of fibres. The simplest are the Voigt and Reuss bounds [8] which allow a framework for effective properties of the equivalent homogeneous material. Hashin and Shtrikman [9] proposed to use a mixed formulation for estimating upper and lower bounds of effective properties of a composite (matrix - inclusions). These bounds are tighter than those of Voigt and Reuss. Kröner [10–12] proposed a self-consistent model from the Eshelby's solution [13] of the problem of consistency in the inclusion. In this model, the inclusion is placed in a medium having the sought effective properties. One of the most used model remains that proposed by Mori and Tanaka in 1973 [14]. It has received wide attention for its simplicity and easiness in applications [15–17]. It takes into account of problem of inclusion of Eshelby, and it's able to also take into account a great number of micro-structural data, associated for instance to the interaction between the reinforcements inside a matrix, without having high computational cost. Use of numerical methods to compute the elastic properties, particularly the finite element analysis, is often devoted to composites with fibres having well defined orientations (Unidirectional, Woven, etc…) [18,19], but rarely for materials with randomly distributed short fibres. Recently, Cunha et al. [20] proposed a numerical approach to simulate the crack behaviour of steel fibre reinforced composites (SFRC),

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using 3D discrete embedded elements for fibres representation. An algorithm based on Monte Carlo method is used to simulate the random fibre distribution over the matrix. Pan and Pelegri [21] presented analytical and numerical tools using finite elements for analysis and design of production grade random chopped fibre composite material.

Motivated by computing the elastic properties of reinforced short natural fibre composites, which requires relatively less computing time, a multi-scale numerical model, named Projected Fibre approach (PF), which uses a special finite element procedure associated with a random distribution of short fibres, is proposed in the present work. It takes into account of the geometry and the mechanical properties of the composite's components. A microscopic truss element is used to model the short fibre reinforcements. The corresponding degrees of freedom are projected on those of the resin matrix. Numerical results of the elastic properties of a reinforced hemp fibre polypropylene composite are compared with those obtained experimentally.

2. The Projected Fibre (PF) approach

2.1. General aspects

The short fibre is considered as an inclusion which could be represented by a 1D truss finite element in a first step. The corresponding stiffness matrix will be projected on that of a 2D finite element associated to the resin space. The approach of Projected Fibres, labelled PF, will use a local condensation of the fibre element degrees of freedom. It may be considered as innovative approach regarding the Mori–Tanaka approach, insofar as it cannot be based on Eshelbi tensors or Euler angles to get the global composite rigidity. The random aspect of the short fibres is represented by the corresponding positions of the truss finite elements. The PF approach has the particularity to consider the elementary short fibre as integrated part of another elementary set named bio-composite, while it is merged inside a resin elementary space. Several parameters may be studied in order to reach a competitive composite. For instance, we refer to geometrical characteristics of the short fibre after the injection process (ratio length/diameter, fibre volume fraction, fibre directions), fibre and resin elastic properties, etc...

2.2. Finite element formulation

2.2.1. 1D Linear truss fibre element

Modelling of one fibre reinforcement by a single discrete truss elastic finite element, with a constant cross section, can be considered as a first hypothesis. This allows associating a fibre to one small cylinder with an average aspect ratio L/D (Length/Diameter). From mechanical point of view, the adopted 2-node fibre element can support an axial load; it can be oriented in any direction within a plane, and then is able to reproduce behaviour of both tensile and compression. No bending load is considered and degrees of freedom are "translations" type defined by a linear approximation of the displacement field u (Fig. 1) which will be detailed in this section. As mentioned before, the cross-sectional dimensions and elastic properties are constant along fibre's length. The linear approximation of global components of the displacement field u, defined in the reference iso-parametric co-ordinate leads to:

$$\begin{pmatrix} U \\ V \end{pmatrix}_{Fibre} = \frac{1-\xi}{2}\begin{pmatrix} U_a \\ V_a \end{pmatrix} + \frac{1+\xi}{2}\begin{pmatrix} U_b \\ V_b \end{pmatrix} \quad (1)$$

Cinematically, the displacement field u (Fig. 1) may be written as a scalar product of director cosines vector $\langle t \rangle$ and the global displacement vector ($\boldsymbol{U}$):

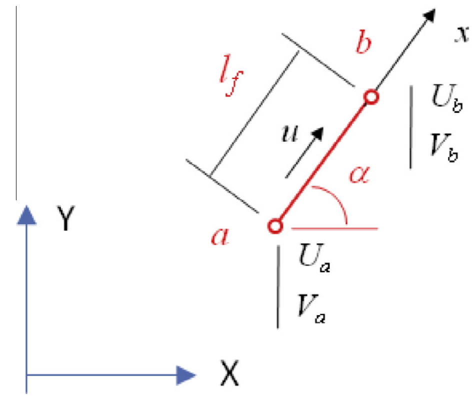

Fig. 1. Truss fibre element (2-node) with orientation α.

$$u = \langle t \rangle (U) = \langle \cos\alpha \quad \sin\alpha \rangle \begin{pmatrix} U \\ V \end{pmatrix} \quad (2)$$

with

$$\cos\alpha = \frac{X_b - X_a}{l_f}; \quad \sin\alpha = \frac{Y_b - Y_a}{l_f} \quad (3)$$

A discrete representation of the following classical elementary internal strain energy

$$\Pi^e_{int}(\text{fibre}) = \frac{1}{2}\langle U_n \rangle [k^e_f](U_n); \quad [k^e_f] \textit{ is the fibre element stiffness matrix} \quad (4)$$

$$\langle U_n \rangle = \langle U_a \quad V_a \quad U_b \quad V_b \rangle : \quad \textit{nodal DOF vector of the fibre element} \quad (5)$$

can lead to the expression of the elementary fibre stiffness matrix $[k^e_f]$

$$[k^e_f] = \frac{E_f A_f}{l_f}[\alpha]; \quad [\alpha] \quad (\textit{fibre orientation matrix}) \quad (6)$$

$$[\alpha] = \begin{bmatrix} [c] & -[c] \\ -[c] & [c] \end{bmatrix}; \quad [c] = \langle t \rangle (t) = \begin{bmatrix} \cos^2\alpha & \cos\alpha \cdot \sin\alpha \\ \cos\alpha \cdot \sin\alpha & \sin^2\alpha \end{bmatrix} \quad (7)$$

E_f, A_f and l_f are respectively the young modulus, the cross-section area and the length of the fibre element. Inside injected samples, the last geometric parameters are represented by microscopic values which can be estimated by using Scanning Electronic Microscope for instance.

2.2.2. Constant strain triangle resin element (CST)

The discrete formulation of the resin element is that of the classical CST (Constant Stain Triangle). It is based on linear Lagrange interpolation (C° continuity) of the displacement field (U,V) (Fig. 2) :

$$\begin{pmatrix} U \\ V \end{pmatrix}_{\text{Re}\sin} = \begin{pmatrix} \lambda U_1 + \xi U_2 + \eta U_3 \\ \lambda V_1 + \xi V_2 + \eta V_3 \end{pmatrix}; \quad \lambda = 1 - \xi - \eta \quad (8)$$

Using the small strains hypothesis, the approximation of the Lagrange deformation field leads to the displacement–strain matrix $[B_r]$:

$$(\varepsilon) = \begin{pmatrix} \varepsilon_X \\ \varepsilon_Y \end{pmatrix} = \begin{pmatrix} \frac{\partial U}{\partial X} \\ \frac{\partial V}{\partial Y} \\ \frac{\partial U}{\partial Y} + \frac{\partial V}{\partial X} \end{pmatrix} = [B_r](U_n) \quad (9)$$

$$(U_n)^T = \langle U_1 \quad V_1 \quad U_2 \quad V_2 \quad U_3 \quad V_3 \rangle \quad (\textit{nodal DOF vector of the resin space}) \quad (10)$$

$$[B_r] = \begin{bmatrix} -j_{11} - j_{12} & 0 & j_{11} & 0 & j_{12} & 0 \\ 0 & -j_{21} - j_{22} & 0 & j_{21} & 0 & j_{22} \\ -j_{21} - j_{22} & -j_{11} - j_{12} & j_{21} & j_{11} & j_{22} & j_{12} \end{bmatrix} \quad (11)$$