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A multi-scale approach for the optimum design of sandwich plates with honeycomb core. Part II: the optimisation strategy

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ABSTRACT

This work deals with the problem of the optimum design of a sandwich panel. The design strategy that we propose is a numerical optimisation procedure that does not make any simplifying assumption to obtain a true global optimum configuration of the system. To face the design of the sandwich structure at both meso and macro scales, we use a two-level optimisation strategy: at the first level we determine the optimal geometry of the unit cell of the core together with the material and geometric parameters of the laminated skins, while at the second level we determine the optimal skins lay-up giving the geometrical and material parameters issued from the first level. The two-level strategy relies both on the use of the polar formalism for the description of the anisotropic behaviour of the laminates and on the use of a genetic algorithm as optimisation tool to perform the solution search. To prove its effectiveness, we apply our strategy to the least-weight design of a sandwich plate, satisfying several constraints: on the first buckling load, on the positive-definiteness of the stiffness tensor of the core, on the ratio between skins and core thickness and on the admissible moduli for the laminated skins.

1. Introduction

Sandwich panels are increasingly used in aerospace, automotive and naval industries thanks to their high stiffness-to-weight and strength-to-weight ratios. In order to have a further weight reduction when employing this kind of structures, in aerospace applications sandwich panels are composed by glass or carbon-fiber composite skins separated by aluminium or resin honeycombs, or by polymer foams. In addition, material and geometrical properties can be designed to provide sandwich plates with different stiffness and density characteristics.

The optimum design of sandwich structures is much more cumbersome than that of a classical monolithic structure. The difficulties increase when the sandwich structure is made of composite skins and a honeycomb core. In this case we have to face, into the same design process, both the difficulty of designing a laminated plate (concerning the skins) and the difficulty of designing a complex 3D cellular continuum such as the honeycomb core. Therefore, engineers always use some simplifying assumptions or rules to obtain, in an easier and faster way, a solution. For example, in [1–3] the optimal design of a sandwich plate is addressed determining exclusively the optimum thickness of both the core and the skins, keeping constant the rest of geometric and material parameters of the system.

Triantafillou and Gibson [4] gave the analytical relations to determine the skins and core thickness and the core density which minimise the weight of a foam core sandwich beam for a given strength. In [5,6] the minimum weight design of sandwich panels under uniaxial compressive loading conditions is solved analytically using the principle of the most efficient chain wherein each link fails simultaneously. The optimisation parameters were the geometry of the unit cell and the thickness of the skins. In this way, also the density of the core is introduced among the design variables. Another analytical study for the minimum weight design of foam-core sandwich panels under stiffness and strength requirements is presented in [7]. Here the three design variables were the thickness of the core and that of the skins (assumed to be identical) along with the core density. A semi-analytical method to minimise the density of truss core structures under prescribed constraints on strength and stiffness is addressed in [8]. However, as it is classical in purely analytical-based approaches, the relations giving rise to the optimal values of the geometric variables are obtained thanks to the imposition of particular load cases (for example uniaxial) and/or boundary conditions (BCs) such as simply supported or

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Notations

BCs	Boundary Conditions
GA	Genetic Algorithm
FE	Finite Element
$\mathbf{A}$	membrane stiffness tensor
$\mathbf{B}$	membrane/bending coupling stiffness tensor
$\mathbf{D}$	bending stiffness tensor
E_i^c	effective Young's moduli of the honeycomb core
G_{ij}^c	effective shear moduli of the honeycomb core
ν_{ij}^c	effective Poisson's ratios of the honeycomb core
h_t, h_b	thickness of the top and bottom skin, respectively
h_c	height of the honeycomb core
l_1	length of the oblique sides of the hexagonal repetitive unit cell
l_2	length of the horizontal sides of the hexagonal repetitive unit cell
t_c	thickness of the foil used to produce the honeycomb core
ϑ	corrugation angle of the hexagonal unit cell
h_{ply}	thickness of the elementary ply
n	number of layers
g_p	optimisation constraints functions
$\mathbf{x}_g$	vector of the geometrical design variables
$\{O; x_1, x_2, x_3\}$	lamina material frame
$\{O; x, y, z\}$	sandwich panel global frame
$\mathbf{Q}$	reduced stiffness tensor of the elementary ply
$T_0, T_1, R_0, R_1, \Phi_0, \Phi_1$	polar parameters of $\mathbf{Q}$
CLPT	Classical Laminate Plate Theory
$\mathbf{N}$	second-rank tensor of membrane forces
$\mathbf{M}$	second-rank tensor of bending moments
$\boldsymbol{\varepsilon}$	second-rank tensor of in-plane strains of the laminate middle plane
$\boldsymbol{\chi}$	second-rank tensor of curvatures of the laminate middle plane
δ_k	fibres orientation angle of the k^{th} ply
h	overall thickness of the generic laminate
$\mathbf{A}^*, \mathbf{B}^*, \mathbf{D}^*$	normalised membrane, membrane/bending coupling and bending stiffness tensors, respectively
$\mathbf{C}$	homogeneity tensor
$T_0^{A^*}, T_1^{A^*}, R_0^{A^*}, R_1^{A^*}, \Phi_0^{A^*}, \Phi_1^{A^*}, K^{A^*}$	polar parameters of $\mathbf{A}^*$
$\mathbf{x}_m$	vector of the mechanical design variables
$\mathbf{x}$	vector of all design variables
W	weight of the sandwich panel
λ	first buckling load of the sandwich panel
λ_{ref}	first buckling load of the reference sandwich panel
$I(f_i)$	objective function for the lay-up design problem
f_i	partial objective functions
DOFs	Degrees Of Freedom
a, b	sandwich panel side lengths along x and y axes, respectively
N_{ind}	number of individuals
N_{gen}	number of generations
p_{cross}	cross-over probability
p_{mut}	mutation probability
ADP	Automatic Dynamic Penalization (method)
E	Young's modulus of the aluminium
ν	Poisson's ratio of the aluminium
ρ	density of the aluminium
E_i	Young's moduli of the carbon-epoxy lamina in the material frame
G_{ij}	shear moduli of the carbon-epoxy lamina in the material frame
ν_{ij}	Poisson's ratios of the carbon-epoxy lamina in the material frame
ρ_s	density of the carbon-epoxy lamina

clamped plate. On the other hand, in the case of more complex BCs it is not possible to obtain an analytical solution, therefore a numerical strategy is needed.

A step further in the optimum design of sandwich panels with corrugated cores was done in [9]. The authors dealt with the problem of the least-weight design of a sandwich plate considering as design variables the thickness of the cell walls as well as that of the skins together with the total height of the panel. They used an analytical model to evaluate both the buckling load of the core and the faces yielding which were considered as optimisation constraints. The optimisation problem was solved using a Genetic Algorithm (GA). Wennhange conducted an interesting work on the weight minimisation of sandwich structures under acoustic constraints. He firstly developed a semi-analytical model in [10], a subsequent experimental verification in [11] and an application to a real-world engineering problem concerning the design of a railway car body in [12]. Other studies on numerical strategies for the optimal design of sandwich structures can be found in [13] for the maximisation of blast load mitigation, in [14] for the optimisation of the head impact mitigation, in [15] where the transverse shear stiffness of the panel is maximised and in [16] for the minimisation of both mass and costs of composite sandwich structures for rail vehicle floor panels.

The objective of the present work is twofold: on one hand, we want to formulate and solve the problem of designing a sandwich panel as an optimisation problem on different scales and, on the other hand, we want to include within the same design process the full set of geometrical and material parameters defining the behaviour of the structure (at each scale) as optimisation variables. In this regard, we propose a very general design strategy that consists in a numerical optimisation procedure without simplifying hypothesis to obtain a true optimal configuration of the system. The design process that we propose is not submitted to restrictions, indeed any parameter characterising our structure is an optimisation variable: the geometry of the unit cell of the core along with the number as well as the orientation angle of the plies for each skin.

In order to deal with the design problem of the sandwich plate at both meso and macro scales, we used a two-level optimisation strategy. At the first level we determine the optimum geometry of the unit cell (core meso-scale) together with the material and geometric parameters of the laminated skins (at this level the laminate representing each skin is modelled as an equivalent homogeneous anisotropic plate whose behaviour at the macro-scale is described in terms of laminate polar parameters, see [17]). At the second level of the strategy, we determine the optimal skins lay-up (the skin meso-scale) meeting the optimal combination of their material and geometrical parameters resulting from the first level of the strategy. The whole procedure is based on one hand on the use of the polar formalism [18] and on the other hand on the new version of the GA BIANCA [19–21].

Since the first level of the strategy involves two different scales (the macro-scale of the sandwich panel as well as the meso-scale of the honeycomb core) we conceived an appropriate model of the repetitive unit cell of the core able to properly evaluate its effective elastic properties used at the macro-scale. This model has been discussed in Part I of the present work. In the first paper we presented the numerical homogenisation technique as well as the related 3D finite element model of the unit cell used within the first level of the optimisation strategy to determine the