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Non-recirculating and recirculating inertial flows of a viscoplastic fluid around a cylinder

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ABSTRACT

Non-recirculating and recirculating inertial flows of a viscoplastic fluid around a cylinder were studied numerically. The Herschel–Bulkley constitutive equation was considered. The flow morphology, the stress and pressure fields around the cylinder and the drag coefficient were determined over a wide range of Reynolds and Oldroyd numbers. The opposing effects of inertia and yield stress on the yielded zones and size of the vortices was demonstrated. Useful formulas in particular for the drag coefficient were also established for engineering purposes. The influence of the power law index was also studied for both shear-thinning and shear-thickening cases and this study revealed a complex behaviour. The position and size of the rigid zones as a function of the power law index and Oldroyd number are represented in different schemes.

1. Introduction

Many industrial systems such as mixers, exchangers or filtration systems involve flows around cylinders or piles of cylinders. A wake, which may be permanent or non-stationary, can sometimes be observed appearing or developing behind these cylinders. This study focuses on the case of viscoplastic fluids and stationary flows. Such fluids are by definition materials that only flow when the stresses applied to them are sufficient to overcome the interaction forces holding them in equilibrium. Once the stress exceeds a threshold τ_0, they are able to flow as a viscous fluid. Viscoplastic fluids are very widely used in the industrial world and are found in our daily environment. They include food fluids, petroleum fluids, cements, geophysical fluids, cosmetics, etc. Their flow is generally governed not only by inertia and viscosity effects, but also by the yield stress and by the shear-thinning index.

Many numerical [1–10] and experimental [11–13] studies have been performed to examine the flows of Newtonian fluids around cylinders in infinite or confined domains [14–18]. Power-law fluids have also been examined in many exclusively numerical studies [19–23].

In cases involving viscoplastic fluids with inertia, Mossaz et al. [24] showed the influence of the Reynolds and Oldroyd numbers and power law index on the criteria governing the appearance of the different regimes such as non-recirculating, recirculating and non-stationary flows with vortex shedding. Refs. [25–27] give details and definitions of the various types of regimes for Newtonian fluids. For cases involving negligible Reynolds numbers, a complete examination can be found in a study performed by Tokpavi et al. [28] and in an article by Putz and Frigaard [29].

While the criteria governing the appearance of the various regimes have recently been determined [24], the characteristics of each regime need to be studied. This study looks at stationary non-recirculating and recirculating regimes.

The study is divided into three parts in order to show the influence of the input parameters on the flow morphology, the distribution of pressures and stresses on the cylinder, and the change in drag coefficient and characteristic lengths:

- The first part demonstrates the successive influence of the Reynolds number, Oldroyd number and power law index on the location of yielded areas and on the lengths that characterise the flow.
- The second part concentrates on changes in the stresses and pressures acting on the cylinder while examining in succession the influence of the three input parameters.
- Finally, the third part demonstrates the successive influence of these three parameters on the drag coefficient.

2. Theory

The problem considered is that of a two-dimensional (2D) flow of an incompressible viscoplastic fluid around a cylinder of diameter

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D. The fluid is assumed to extend to infinity. The velocity of the fluid at infinity is designated $V\infty$ and it is assumed that the fluid sticks to the cylinder wall. The dimensions are represented on Fig. 1.

The mass and momentum conservation equations for this fluid may be written as follows:

$$\text{Mass conservation}: \quad \nabla \cdot \underline{u} = 0 \tag{1}$$

$$\text{Momentum conservation}: \quad \rho\left(\frac{\partial \underline{u}}{\partial t} + \underline{u} \cdot \nabla \underline{u}\right) - \nabla \cdot \underline{\underline{\sigma}} = 0 \tag{2}$$

$$\text{with}: \quad \underline{\underline{\sigma}} = -p\underline{\underline{I}} + \underline{\underline{\tau}} \tag{3}$$

where ρ is the density, u the velocity vector with components U_x and U_y, p the pressure, $\underline{\underline{I}}$ the indentity matrix, $\underline{\underline{\sigma}}$ the stress tensor and $\underline{\underline{\tau}}$ is its deviatoric part.

Viscoplastic fluids modelled by the Herschel–Bulkley law are considered.

Therefore:

$$\begin{cases} \underline{\underline{\tau}} = \left(K\dot{\gamma}^{(n-1)} + \frac{\tau_0}{\dot{\gamma}}\right)\underline{\underline{\dot{\gamma}}} & \text{if} \quad \tau > \tau_0 \\ \underline{\underline{\dot{\gamma}}} = 0 & \text{if} \quad \tau \leqslant \tau_0 \end{cases} \tag{4}$$

with

$$\dot{\gamma}_{ij} = \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \tag{5}$$

where n is the power law index, K the consistency, τ_0 the yield stress, $\underline{\underline{\dot{\gamma}}}$ the strain rate tensor, $\dot{\gamma} = \sqrt{\frac{1}{2}tr(\underline{\underline{\dot{\gamma}}}^2)}$ the shear rate and $\tau = \sqrt{\frac{1}{2}tr(\underline{\underline{\tau}}^2)}$ the magnitude of the stress tensor.

The boundary conditions for the problem may be written:

- at the inlet : $U_x = V\infty$ and $U_y = 0$,
- on the cylinder : $V = 0$ (adherence),
- at the outlet : outflow condition (a zero flux diffusion for all flow variables).

The flow may be characterised by three basic dimensionless numbers:

- *The power law index: n.*
- *The Oldroyd number:* this represents the ratio of plastic effects to viscous effects. For a fluid with a Herschel–Bulkley constitutive equation, it is defined by:

$$Od = \frac{\tau_0}{K(V_\infty/D)^n} \tag{6}$$

- *The Reynolds number:* this represents the ratio of inertial effects to viscous effects. For a fluid with a Herschel–Bulkley constitutive equation, it is defined by:

$$Re = \frac{\rho V_\infty^{2-n} D^n}{K} \tag{7}$$

Other useful dimensionless parameters have been defined:

- *The plastic Reynolds number*: this represents the ratio of inertial effects to total shear stress effects. This number is defined as follows:

$$Rep = Re/(1 + Od) \tag{8}$$

- *Critical numbers:* These critical numbers correspond to changes in morphology within the flow. The first ones (Re_c, Od_c, Rep_c) represent the limit between no recirculating and recirculating flows. In the same way Re^c, Od^c and Rep^c define the limit between symmetrical recirculating flow and asymmetrical flow with vortex shedding. They thus represent the start of inertial instability with vortex shedding. The values of these limits for a Bingham fluid ($n = 1$) were calculated by Mossaz et al. [24]. The following laws were found:

$$Re_c = 48.3\ Od_c + 7 \tag{9}$$

$$Re^c = 45.8\ Od^c + 47 \text{ or } Rep^c \approx 47 \tag{10}$$

- The *drag coefficient Cd* is defined by:

$$Cd = \frac{2F_d}{\rho V_\infty^2 D} \tag{11}$$

where F_d is the drag force per unit length calculated on the cylinder.

- *Characteristic length L_1*: this dimensionless length is defined by $L_1 = (Lw - 0.5D)/D$ with Lw being the length of the recirculation calculated from the velocity field horizontally along the x-axis in the recirculating regime (Fig. 2).
- *Characteristic length L_2*: this dimensionless length is defined by $L_2 = (Lr - 0.5D)/D$ with Lr being the length of the static rigid zone downstream of the cylinder (Fig. 2). Lr is calculated using the change in τ, the magnitude of the stress tensor, along the horizontal axis.
- *Angle of separation θ_c*: this is the angle at which the streamlines separate from the cylinder (Fig. 2c and d). θ_c is calculated using the change in τ on the cylinder.

3. Numerical method

In order to avoid the discontinuity in the Herschel–Bulkley constitutive equation, it is regularised by using Papanastasiou's modification [30]. This model has been used in numerous studies [30–34]

$$\underline{\underline{\tau}} = \left(K\dot{\gamma}^{(n-1)} + \frac{\tau_0(1 - \exp(-M\dot{\gamma}))}{\dot{\gamma}}\right)\underline{\underline{\dot{\gamma}}} \tag{12}$$

M represents the regularisation parameter. Thereafter, $m = MD/U$ will represent the non-dimensionalised form of M.

The Ansys-Fluent software [35] (version 6.2.16) was used for this study. It is based on Finite Volume Method. A refined grid with quadrilateral elements was defined around the cylinder in order to take into account the problem of the boundary layer. For the numerical method, more details can be found in the previous article [24]. But it can specified that the two dimensional, laminar, steady solver of Ansys-Fluent was used to solve the incompressible flow around a cylinder. The second order upwind scheme has been used to discretize the convective terms in the momentum equations because the velocity field is complex and the flow crosses the meshes obliquely. The conduction terms are calculated to the second order The "semi-implicit-consistent" method is used for solving the pressure-velocity coupling.

In this study, the boundary between flowing and rigid zones is obtained by using the condition $\tau = \tau_0\ (1 \pm \varepsilon)$ [36], where ε is a small number dependent on the configuration and regularisation parameter m. All zones where the inequality $\tau > \tau_0\ (1 \pm \varepsilon)$ is verified are considered to be flowing; if not they are considered to be rigid. The criterion $\varepsilon = 0$ was used in this study.

To be able to define the rigid zones precisely [24], a convergence of 10^{-13} was used in this study for the residue of the velocity, and for the residue of the mass conservation equation.