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A numerical solution for vibration analysis of composite laminated conical, cylindrical shell and annular plate structures

Xie Xiang *, Jin Guoyong, Li Wanyou, Liu Zhigang

College of Power and Energy Engineering, Harbin Engineering University, Harbin 150001, PR China

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ABSTRACT

This paper focuses on the free vibration analysis of composite laminated conical, cylindrical shells and annular plates with various boundary conditions based on the first order shear deformation theory, using the Haar wavelet discretization method. The equations of motion are derived by applying the Hamilton's principle. The displacement and rotation fields are expressed as products of Fourier series for the circumferential direction and Haar wavelet series and their integral along the meridional direction. The constants appearing from the integrating process are determined by boundary conditions, and thus the equations of motion as well as the boundary condition equations are transformed into a set of algebraic equations. Then natural frequencies of the laminated shells are obtained by solving algebraic equations. Accuracy, stability and reliability of the current method are validated by comparing the present results with those in the literature and very good agreement is observed. Effects of some geometrical and material parameters on the natural frequencies of composite shells are discussed and some representative mode shapes are given for illustrative purposes. Some new results for laminated shells are presented, which may serve as benchmark solutions.

1. Introduction

Composite laminated shell structures are widely used in a variety of engineering applications, such as the aerospace, shipping, military, and other industries. The increasing use of composite shell structures has motivated great interest in developing various mathematical models and computational methods for analyzing their dynamic behaviors. Transverse effects become more pronounced as the shell becomes thicker relative to its in-plane dimension and radius of curvature. Hence, considering the shear deformation is essential in the study of moderately thick shell structures. Since the classical shell theory (CSTs) [1–5] is based on the Kirchhoff–Love assumptions, in which transverse normal and shear deformations are neglected, developing other theories for moderately thick shells have been a major issue. So far, first-order shear deformation theory (FSDTs) and higher-order shear deformation theory (HSDTs) have been proposed and developed. Since the transverse shear strains in the FSDTs [6–9] are assumed to be constant in the thickness direction, shear correction factor has to be incorporated to adjust the transverse shear stiffness for practical shell problems. To avoid the use of shear correction factor and have a better prediction of the vibration behavior of moderately thick shells, a number of HSDTs [10–14] have been proposed. However, as pointed out by Qu et al. [7], these HSDTs are computationally more demanding than those FSDTs, and moreover they introduce rather sophisticated formulations and boundary terms that are not easily applicable or yet understood. Therefore, a careful selection of the appropriate shell theory is decisive for free vibration analysis of composite laminated shells. From the existing literature, we can know that the FSDT with proper shear correction factor is adequate for the prediction of the global behaviors of moderately thick shells. With this in mind, the FSDT is just employed in the present analysis.

Apart from the aforementioned shear deformation theories, it has also been of great interest for researchers to develop accurate and efficient methods which can be used to determine the vibration behaviors of composite laminated shells. Although substantial descriptions of various methods for vibration analysis of composite shells are available in the review articles [15–17] and monographs [1,18,19], a brief introduce of recent works relating to the vibration analysis of laminated shells of revolution, including the conical, cylindrical shells and annular plates, is still necessary. So far, most of the studies are about laminated cylindrical shell. Various methods have been proposed and developed to handle the free vibration problems of such shells, such as the Rayleigh–Ritz method, Galerkin's method, and finite element method. As is expected, the study on the vibration of cylindrical shells should be naturally progressed to that of conical shell, and yet the conical coordinate system is function of the meridional coordinate, the resulting equations of

* Corresponding author. Tel.: +86 451 82589199.
E-mail address: xiexiangFQ@126.com (X. Xiang).

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motion for laminated conical shells consist of a system of partial differential equations with variable coefficients. This involves the inherent complexity for solving the equations of motion for conical shells, and the implementation of closed-form approach is restricted [7]. A few scholars have made efforts to deal with vibration problems of this types of shell structures with approximate analytical and numerical techniques, such as differential quadrature (DQ) method [2,4,9], discrete singular convolution (DSC) method [5,6], a general domain decomposition method [7], spline method [8], meshless method [13,17], and finite element method [20]. In addition, a few selected works related to the free vibrations of laminated annular plates have been devoted by Narita [21], Lin and Tseng [22], Vera et al. [23]. From the review of the literature, it shows that despite various methods for vibration analysis of composite shell structures, finding reliable and efficient approaches for laminated shells of revolution with different boundary condition is still a big challenge. Therefore, the purpose of the present work is to introduce a simple yet powerful method for the free vibration analysis of the considered structures. The solution is obtained by using the numerical technique termed the Haar wavelet discretization method, which leads to a generalized eigenvalue problem. The main features of the numerical technique are described in Section 2. Mathematical fundamentals and recent developments of the present method as well as its applications in engineering are stated below in detail. With good features in treating singularities, the Haar wavelet series has been used to solve the vibrations of functionally graded plate in Ref. [24] and the damage evaluation of plates in Ref. [25]. Majak et al. [26] developed this method and introduced it for solving solid mechanics problems. In Ref. [27], Majak discussed the strong and weak formulations, and pointed out the weak formulation based Haar wavelet discretization method (HWDM) is more efficient and stability in the case of large number of collocation points. Recently, Hein and Feklistova [28,29] based on Haar wavelet series solved the vibrations of non-uniform and functionally graded beams with various boundary conditions. The present works can be considered as an extension of the authors' previous works [30–32] to consider the effects of transverse shear deformation and rotary inertia for composite laminated shells. In order to verify the convergence, efficiency and accuracy of present method, free vibrations of cross-ply and angle-ply laminated conical, cylindrical shells and annular plates are investigated with different geometric and material parameters. The main aim of this present paper is to demonstrate a convenient and efficient application of the Haar wavelet discretization method to the free vibrations of composite laminated shells and provide a simple yet powerful alternative to other analytical and numerical techniques.

2. Theoretical formulations

2.1. The Haar wavelet series and their integrals

For the sake of completeness, some aspects related to the Haar wavelet will be described. The orthogonal set of Haar wavelet $h_i(\xi)$ is a group of square waves with magnitude of ±1 in some intervals and zeros elsewhere [26]

$$h_i(\xi) = \begin{cases} 1 & \xi \in [\xi^{(1)}, \xi^{(2)}] \\ -1 & \xi \in [\xi^{(2)}, \xi^{(3)}] \\ 0 & elsewhere \end{cases} \tag{1}$$

where the notations $\xi^{(1)} = \frac{k}{m}$, $\xi^{(2)} = \frac{k+0.5}{m}$, $\xi^{(3)} = \frac{k+1}{m}$ are introduced. The integer $m = 2^j (j = 0, 1, \ldots, J)$ indicates the level of the wavelet, where J is the maximal level of resolution. $k = 0, 1, \ldots, m-1$ is the translation parameter. The subscript i can be expressed as $i = m + k + 1$, in the case $m = 1$, $k = 0$ we have $i = 2$; the maximal value of i is $i = 2M = 2^{J+1}$. The case $i = 1$ corresponds to the scaling function: $h_1(\xi) = 1$ for $\xi \in [0,1]$ and $h_1(\xi) = 0$ elsewhere.

Any function $y(x)$, which is square integrable in the interval [0,1], can be expanded into Haar wavelet series of infinite terms. If $y(x)$ is piecewise constant by itself, or may be approximated as piecewise constant during each subinterval, then $y(x)$ will be truncated with finite terms, that is

$$y(x) = \sum_{i=1}^{2M} a_i h_i(x) \tag{2}$$

where a_i ($i = 1, \ldots, 2M$) is unknown wavelet coefficient. The interval [0,1] is divided into $2M$ subintervals of equal length $\Delta x = 1/2M$; the collocation points are given as:

$$\xi_l = \frac{(l-0.5)}{2M}, \quad l = 1, 2, \ldots, 2M \tag{3}$$

The Haar coefficient matrix $\boldsymbol{H}$ is defined as $\boldsymbol{H}(i,l) = h_i(\xi_l)$. If we want to solve an nth order PDE, the following integrals are required [33]

$$p_{\alpha,i}(x) = \underbrace{\int_0^{\xi}\int_0^{\xi}\cdots\int_0^{\xi}}_{\alpha-times} h_i(t)dt^{\alpha} = \frac{1}{(\alpha-1)!}\int_0^{\xi}(x-t)^{\alpha-1}h_i(t)dt$$
$$\alpha = 1, 2, \ldots, \quad n, i = 1, 2, \ldots, 2M. \tag{4}$$

The case $\alpha = 0$ corresponds to the function $h_i(t)$. These integrals can be calculated analytically. In the case $i = 1$, we have $p_{\alpha,i}(\xi) = \xi^{\alpha}/\alpha!$; and in the case $i > 1$ we obtain the integrals as follows [28]:

$$p_{n,i}(\xi) = \begin{cases} 0 & \xi < \xi^{(1)} \\ \frac{1}{n!}(\xi-\xi^{(1)})^n & \xi^{(1)} < \xi < \xi^{(2)} \\ \frac{1}{n!}[(\xi-\xi^{(1)})^n - 2(\xi-\xi^{(2)})^n] & \xi^{(2)} < \xi < \xi^{(3)} \\ \frac{1}{n!}[(\xi-\xi^{(1)})^n - 2(\xi-\xi^{(2)})^n + (\xi-\xi^{(3)})^n] & \xi > \xi^{(3)} \end{cases} \tag{5}$$

For solving boundary value problems, the values $p_{\alpha,i}(0)$ and $p_{\alpha,i}(1)$ should be calculated in order to satisfy the boundary conditions. Substituting the collocation points in Eq. (3) into Eq. (5) yields

$$\boldsymbol{P}^{(\alpha)}(i,l) = p_{\alpha,i}(\xi) \tag{6}$$

where $\boldsymbol{P}^{(a)}$ is a $2M \times 2M$ matrix. It should be noted that calculations of the matrices $\boldsymbol{H}(i,l)$ and $\boldsymbol{P}^{(\alpha)}(i,l)$ must be carried out only once.

2.2. Geometrical configuration

Consider a composite laminated shell of revolution with an arbitrary number of layers, which are perfectly bonded together. The geometric parameters and coordinate system of a differential element of a laminated shell are shown in Figs. 1 and 2. From Fig. 1, the reference surface of the shell is taken to be at its middle surface where an orthogonal curvilinear coordinate system (x,s,z) is fixed. The total thickness of the shell is h. The included angle between the material coordinate of the kth layer and the x-axis of the structure is denoted by θ, and the index k denotes the layer number which starts from the shell bottom. The displacement of the shell in the x, s and z directions are denoted by u, v and w, respectively. In Fig. 2(b), the geometry and notation for the coordinates are shown. The cone length and cone semi-vertex angle of the shell are represented by L and α, respectively. R_1 and R_2 are the radius of the cone at its small and large edges. The radius R is a function of axial coordinate x and $\varphi = \pi/2 - \alpha$ is the angle between the normal of the shell surface z and the axis $\boldsymbol{o}$. In Fig. 2(a), it is worth noting that, by setting the semi-vertex angle $\alpha = 0$, we can reduce the formulation of conical shells to that of cylindrical shells. In