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Multi-material topology optimization of laminated composite beams with eigenfrequency constraints

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ABSTRACT

This paper describes a methodology for simultaneous topology and material optimization in optimal design of laminated composite beams with eigenfrequency constraints. The structural response is analyzed using beam finite elements. The beam sectional properties are evaluated using a finite element based cross section analysis tool which is able to account for effects stemming from material anisotropy and inhomogeneity in sections of arbitrary geometry. The optimization is performed within a multi-material topology optimization framework where the continuous design variables represent the volume fractions of different candidate materials at each point in the cross section. An approach based on the Kreisselmeier–Steinhauser function is proposed to deal with the non-differentiability issues typically encountered when dealing with eigenfrequency constraints. The framework is applied to the optimal design of a laminated composite cantilever beam with constant cross section. Solutions are presented for problems dealing with the maximization of the minimum eigenfrequency and maximization of the gap between consecutive eigenfrequencies with constraints on the weight and shear center position. The results suggest that the devised methodology is suitable for simultaneous optimization of the cross section topology and material properties in design of beams with eigenfrequency constraints.

1. Introduction

A typical objective in the design of flexible structures subjected to dynamic loads concerns the maximization of the minimum eigenfrequency or the maximization of the gap between consecutive eigenfrequencies. From the many different methodologies proposed in the literature, topology optimization techniques have proved a promising alternative. Diaz and Kikuchi [1] and Ma et al. [2] presented results for structural topology optimization of two-dimensional structures. Pedersen [3] and Du and Olhoff [4] addressed the problem concerning the control of the dynamic properties of plates. Luo and Gea [5] and Gea and Luo [6] presented a strategy for optimizing the location and orientation of stiffeners for eigenfrequency placement design of shell structures. Furthermore, Stegmann and Lund [7] and Pedersen [8] have presented solutions for the maximization of the minimum eigenfrequency design of laminated composite plates. The optimal design of beams with eigenfrequency constraints, however, has mostly concerned two dimensional problems addressing only the optimization of the cross section dimensions along the beam length (see, e.g., Olhoff [9] and Bendsøe and Olhoff [10]).

An extension of the computational framework suggested by Blasques and Stolpe [11] combining a high-fidelity beam model and multi-material topology optimization techniques, is presented here to include eigenfrequency constraints. Preliminary results are presented in which the cross section topology and laminate properties of prismatic cantilevered laminated composite beams are optimized simultaneously. It is shown that the framework is suitable for eigenfrequency tailoring of a general class of beam-like structures. Potential applications include aeroelastic optimization of wind turbine blades for mitigation of aeroelastic instabilities, among other. To the author's best knowledge no previous publication addresses the simultaneous topology and material optimization of beam cross sections with eigenfrequency constraints as presented here.

The proposed framework relies on a high-fidelity beam finite element model for the analysis of the structural response. These type of modeling approach allows for a computationally inexpensive representation of three dimensional beam-like structures. The global response of the beam – e.g., compliance and eigenfrequencies – can be determined with great accuracy using a model which is computationally much less costly than its three-dimensional shell or solid finite element counterparts. This capability has been exploited in computationally intensive applications, e.g., wind turbine aeroelastic simulation tools (see, e.g., Larsen and Hansen [12]). The generation of the beam model is divided in

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two parts. The first and most challenging part concerns the solution of a two-dimensional problem dealing with the determination of the cross section stiffness and mass properties. In the second part, the previously computed cross section properties are integrated along the beam length to obtain the beam finite element stiffness and mass matrices. The sectional properties are analyzed here using the BEam Cross section Analysis Software (BECAS), an open-source implementation by Blasques and Lazarov [13] of the original theory by Giavotto et al. [14]. BECAS is a finite element based tool which is able to account for the effects of material anisotropy and inhomogeneity in the analysis of the stiffness and mass properties of beam sections of arbitrary geometry. The reader is referred to Jung et al. [15], Volovoi et al. [16], and the comprehensive work by Hodges [17] for a review on different beam modeling techniques.

In this context, the optimal design problem concerns the distribution of a limited amount of different materials within a design domain represented here by the cross section finite element mesh. A change in the material distribution in the cross section results in a consequent change of its stiffness and mass properties and in turn, of the structural response of the beam. This optimal design problem is solved using the multi-material topology optimization framework presented by Blasques and Stolpe [11], Hvejsel and Lund [18], and Hvejsel et al. [19]. The framework is based on the principles of topology optimization (see, e.g., Bendsøe and Sigmund [20]) and relies on extensions to include multiple anisotropic materials of the Solid Isotropic Material with Penalization (SIMP) material interpolation technique (Bendsøe and Kikuchi [21] and Rozvany and Zhou [22]), and the density filtering scheme by Bruns and Tortorelli [23]. This approach is a variation of the so-called discrete material optimization technique originally presented by Lund and Stegmann [24] and Stegmann and Lund [7] and applied to the optimal design of laminated composite shell structures.

A common issue when dealing with eigenfrequency constraints concerns the fact that the order of the eigenfrequencies may change throughout the optimization procedure. This will in turn lead to non-differentiability and consequently to a non-robust convergence behavior of methods for smooth optimization, namely, gradient-based methods. A typical approach to mitigate these effects consists of applying the so-called bound formulation (see, e.g., Bendsøe and Sigmund [20]). An alternative approach is proposed here using the Kreisselmeier–Steinhauser (KS) function (Kreisselmeier and Steinhauser [25]) to approximate the maximum and minimum values of groups of eigenfrequencies. The KS function is a continuously differentiable envelope function which approximates the maximum or minimum of a set of functions. The functions should be continuous but need not be continuously differentiable. The aim is to try to improve the convergence behavior by rewriting the eigenfrequency constraints to take advantage of the mathematical properties of the KS function. The mathematical properties of the KS function have been discussed by Raspanti et al. [26]. Moreover, it has been used in similar optimal structural design contexts as a constraint aggregation function by, e.g., Martins et al. [27] and Maute et al. [28].

The paper is organized as follows. The beam finite element structural model is briefly described in Section 2. The multi-material topology optimization framework and problem formulations are described in Section 3, where the KS function is also presented. The gradients or sensitivities for each of the objective functions and constraints are presented in Section 4. Section 5 describes the setup of the numerical experiments, presents the optimized cross section designs, and discusses the results. Finally, the most important conclusions of the work presented in this paper are summarized in Section 6.

2. Structural model

The structural response of the beam is analyzed based on the beam finite element model presented by Blasques and Stolpe [11]. The model is extended here for the analysis of the beam eigenfrequencies and eigenmodes.

When using beam models it is assumed that the original beam structure is represented by a reference line along the length of the beam going through the reference points of a given number of representative cross sections. The two steps involved in the generation of the beam model are discussed next. The first step concerns the evaluation of the cross section stiffness and mass properties as discussed in Section 2.1. The second part concerns the integration of these properties to generate the beam finite elements. The latter is addressed in Section 2.2 where the derivation of the beam finite element stiffness and mass matrices is presented along with the equations of motion for the analysis of the dynamic response of the beam.

2.1. Cross section analysis

For a linear elastic beam there exists a linear relation between the cross section generalized forces $\mathbf{T}$ and moments $\mathbf{M}$ in $\boldsymbol{\theta} = [\mathbf{T}^T\mathbf{M}^T]^T$, and the resulting strains $\boldsymbol{\tau}$ and curvatures $\boldsymbol{\kappa}$ in $\boldsymbol{\psi} = [\boldsymbol{\tau}^T\boldsymbol{\kappa}^T]^T$ (see Fig. 1). This relation is given in its stiffness form as $\mathbf{K}_s\boldsymbol{\psi} = \boldsymbol{\theta}$, where $\mathbf{K}_s$ is the 6×6 cross section stiffness matrix. In the most general case, considering material anisotropy and inhomogeneity, all the 21 stiffness parameters in $\mathbf{K}_s$ may be required to describe the deformation of the cross section. In the current research, the entries of $\mathbf{K}_s$ are determined using the BEam Cross section Analysis Software (BECAS), an implementation by Blasques and Lazarov [13] of the theory by Giavotto et al. [14]. The formulation relies on a finite element discretization of the cross section to approximate the cross section in-plane and out-of-plane deformation or warping. BECAS is able to estimate the stiffness properties of beam sections with arbitrary geometry and correctly account for the effects stemming from material anisotropy and inhomogeneity. A brief outline of the theory underlying the determination of $\mathbf{K}_s$ is presented here. The reader is referred to Blasques and Stolpe [11] for more details on the derivation and notation.

The determination of $\mathbf{K}_s$ entails the solution to a two-dimensional problem associated with the determination of three-dimensional deformation of the cross section. The solution is obtained from the cross section equilibrium equations given by the following system of linear equations

$$\mathbf{KW} = \mathbf{F} \tag{1}$$

where he coefficients in matrix $\mathbf{K}$ are associated with the stiffness of the cross section. Furthermore, the solution matrix $\mathbf{W}$ contains the cross section rigid body motions $\boldsymbol{\psi}$ and the three dimensional warping displacements $\mathbf{u}$. Finally, the load array $\mathbf{F}$ is associated with a series of unit load vectors $\boldsymbol{\theta}$. The solution $\mathbf{W}$ from (1) is

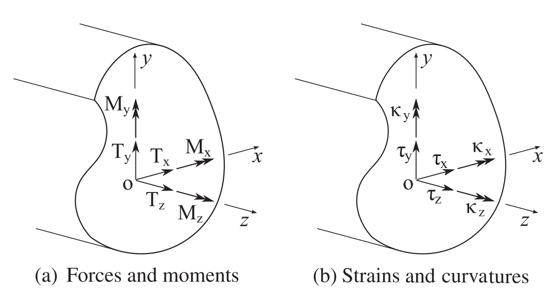

Fig. 1. Cross section coordinate system, foces and moments (a), and corresponding strains and curvatures (b).