Composite Structures 111 (2014) 130–137

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

Experimental validation of the modified couple stress Timoshenko beam theory for web-core sandwich panels

Jani Romanoff [a], J.N. Reddy [b,*]

[a] Aalto University, School of Engineering, Department of Applied Mechanics, Tietotie 1C, 00076 Aalto, Finland
[b] Texas A&M University, Mechanical Engineering, College Station, TX 77843-3123, USA

ARTICLE INFO

Article history:
Available online 18 December 2013

Keywords:
Couple stress
Timoshenko beam
Homogenization
Web-core sandwich
Experimental validation

ABSTRACT

The paper presents experimental validation of the modified couple stress Timoshenko beam theory for web-core sandwich panels. The face and web-plates are assumed to be isotropic and to behave according to the kinematics of the Euler–Bernoulli beam theory. First, a modified couple stress theory for Timoshenko beams is reviewed. Then shear, bending and couple stress responses of homogenized web-core sandwich beams are examined. The developed theory is validated with experiments from open literature for beams in 3- and 4-point bending. The beams have 4, 9 and 15 unit cells along their length. It is seen that the developed theory is in excellent agreement with experiments. It is also seen that the theory converges to physically correct solutions in the cases of infinite and zero shear stiffness; while the first corresponds the case of Euler–Bernoulli sandwich beam, the second corresponds the case where the sandwich effect is lost and the bending is carried out purely by the bending of the faces. The paper also gives explicit expression for the couple stress stiffness in terms of unit cell dimensions and materials.

1. Introduction

The increasing demand for safe, sustainable and environmentally acceptable structures has increased the need to investigate new structural configurations. Sandwich panels with periodic, unidirectional, core are one group of many available alternatives. Due to the fact that face plates are far from neutral axis, these panels are efficient in bending; see Fig. 1 and Refs. [1,2]. The unidirectional core makes the panels attractive to systems integration such as cabling. Typical materials in these panels are wood, cardboard, GFRP, steel or aluminum and these are produced by pultrution, lamination, adhesive bonding, friction-stir- or laser-welding; see Refs. [3–15]. These panels, however, suffer from high orthotrophy, especially in out-of-plane shear, which makes them susceptible for large shear deformations, i.e. warping, opposite to the core direction; see Refs. [10,13,16–25].

Traditional continuous core sandwich panels are based on the load carrying mechanism where faces carry the global bending moment and the core the shear forces; see for example Refs. [1,2,17,21]. In sandwich panels with unidirectional core this load carrying becomes mixed, i.e. also the faceplates participate to the load-carrying of shear forces; see Refs. [10,13,17,21–25] for various core types. This means that when there is no filling material within the unit cells, the face and core plate bending carries the entire shear force opposite to the core direction. This means that the unit cell response is complex phenomena with core and face plates interacting with each other and causing considerable stresses at the unit cell level; see Refs. [13,22–24]. Thus, in order to capture these stresses the structural analysis must consider the unit cell response together with the global response. However, in practice the structures can be large can contain numerous unit cells. Therefore, homogenization is often used. It, however, deals with average response mapped into equivalent continuum; see Refs. [23,24,26–29].

Fundamental questions in using the equivalent continuum are the lowest number of unit cells that one can consider and which kind of continuum is sufficient. Recently, several papers have been written on the couple stress based beam theories based on the works from Refs. [30–33]. These papers extend the classical Euler–Bernoulli and Timoshenko beam theories to account micro-structural effects within the continuum; see for example Refs. [33–39]. These extensions have been carried out for geometrically non-linear problems using von Kármán strains [35], for bifurcation buckling and vibrations and for plates; see Refs. [40–44]. However, the papers model the micro-structural effects though l^2-parameter, which is related to the micro-structural characteristic length of the problem in hand. The characteristic length can be related to the wavelength in vibration and buckling, or for example to thickness of the composite layer; see Ref. [41]. In web-core sandwich beams the bending response is well described in terms of periodic and homogenized response in Refs. [22–24]; and for more extensive

* Corresponding author. Tel.: +1 9798622417.
E-mail address: jnreddy@tamu.edu (J.N. Reddy).

0263-8223/$ - see front matter
http://dx.doi.org/10.1016/j.compstruct.2013.11.029

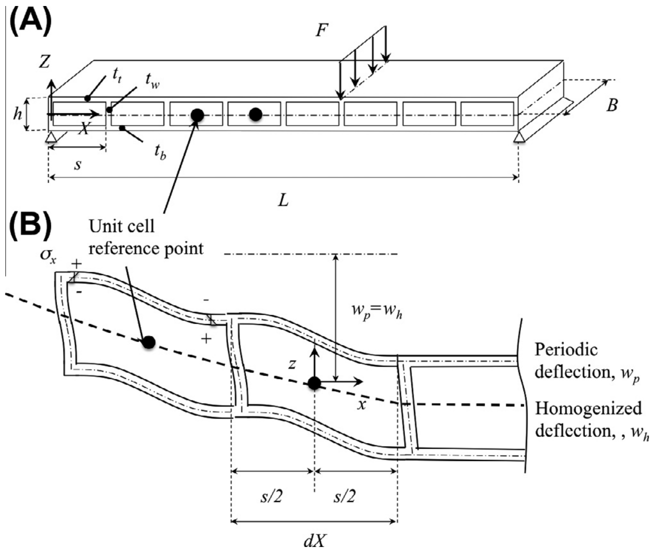

Fig. 1. Periodic and homogenized bending deformations in web-core sandwich beam.

truss-like microstructures in Ref. [29]. To the best of authors' knowledge, the couple stress theory has not been applied to web-core sandwich panels where the actual local response of the microstructure can be easily described using analytical formulations.

Thus, the aim of this paper is to investigate the bending of web-core sandwich beams using the modified couple stress theory of homogenous Timoshenko beams as formulated by Ma et al., [34] and Reddy [35]. First, a brief review of the theory is given. Then, the homogenized bending response of web-core sandwich beams by Romanoff, in Refs. [22,23] is presented that accounts the effects of the periodicity in the beams. The two methods are combined to give the average response of the web-core sandwich beams in terms of deflections. The theory is validated with three sets of experiments where the unit cell size to length of beam ratio is varied from $l/L = 1/4–1/15$. The experiments are reported for 3- and 4-point bending in Refs. [45,25,46] respectively.

2. Definitions and assumptions

The sandwich beam is assumed to consist of small structural elements representing the web and the faceplates, which deform by both in-plane and bending loads. For simplicity the local deformation of these elements is assumed to happen according to Euler–Bernoulli beam theory. The core of the sandwich beam is considered to be between the faceplates and it includes both the web plates and the voids between them. Two coordinate systems are used, the global XYZ, located at the mid-plane of the sandwich beam and the local, located at the mid-point of each unit cell; see Fig. 1. The web plates are in the YZ-plane and have a thickness t_w and a height h_c. The web plate spacing is denoted by s, which is also the unit cell length. The face plates are in the XY-plane and have a thickness t. The voids between the face and web plates are empty. Subscripts t, b and w are used for the top face, the bottom face and the web plates, respectively. The plate has length L and breadth B, total height $h = t_t + t_b + h_c$ and the neutral axes of the faceplates have a distance of $d = (t_t + t_b)/2 + h_c$. Notations F and M are used for the point forces and point moments per unit breadth. The Young's modulus, shear modulus and Poisson's ratio are denoted with E, G and ν, respectively.

3. The modified couple stress Timoshenko beam theory of Ma, Gao, and Reddy

3.1. Derivation of the differential equations

The equations governing the bending of homogenous beams are derived assuming Timoshenko beam theory; see Fig. 2 and Refs. [34,35]. Only the essential parts of the derivation are reviewed here.

According to principle of virtual displacements, the virtual strain energy is given as [35]

$$\delta U = \int_0^L \int_A (\sigma_{ij}\delta\varepsilon_{ij} + m_{ij}\chi_{ij})dAdx \tag{1}$$

where summation of repeated indices is implied. Here m_{ij} is the deviatoric part of the couple stress tensor and the corresponding curvature tensor is given as [35]

$$\begin{aligned} &\omega_1 = 0,\ \omega_2 = \frac{1}{2}\left(\frac{\partial u}{\partial Z} - \frac{\partial w}{\partial X}\right),\ \omega_3 = 0 \\ &\chi_{12} = \frac{1}{2}\left(\frac{\partial \omega_2}{\partial X_1}\right) = \frac{1}{4}\left(\frac{\partial}{\partial X}\frac{\partial u}{\partial Z} - \frac{\partial^2 w}{\partial X^2}\right) \end{aligned} \tag{2}$$

where according to Timoshenko beam theory,

$$\begin{aligned} &u_X(X,Z) = u(X) + Z\phi_x(X) \\ &w(X,Z) = w(X) \end{aligned} \tag{3}$$

and further

$$\frac{\partial u}{\partial Z} = \phi_x \tag{4}$$

Thus, the strains are given as

$$\begin{aligned} \varepsilon_{xx} &= \frac{\partial u}{\partial X} + Z\frac{\partial \phi_x}{\partial X},\quad \gamma_{xz} = \phi_x + \frac{\partial w}{\partial X},\quad \chi_{12} = \frac{1}{4}\left(\frac{\partial \phi_x}{\partial X} - \frac{\partial^2 w}{\partial X^2}\right) \\ &= \frac{1}{4}\left(\frac{\partial \gamma_{xz}}{\partial X} - 2\frac{\partial^2 w}{\partial X^2}\right) \end{aligned} \tag{5}$$

The stress resultant are

$$\begin{aligned} &N_{xx} = \int_A \sigma_{xx} dA,\quad M_{xx} = \int_A \sigma_{xx} Z dA,\quad Q_{xz} = \int_A \sigma_{xz} dA, \\ &P_{xy} = \int_A m_{xy} dA \end{aligned} \tag{6}$$

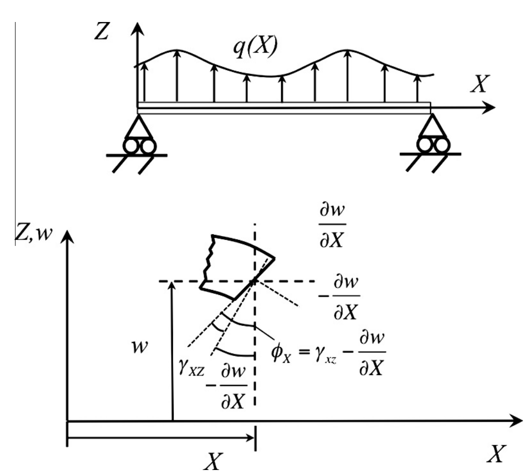

Fig. 2. Loading and kinematics of the Timoshenko beam.