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Static and dynamic of carbon nanotube reinforced functionally graded cylindrical panels

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ABSTRACT

The analysis of flexural strength and free vibration of carbon nanotube reinforced composite cylindrical panels is carried out. Four types of distributions of uniaxially aligned reinforcements are considered, i.e. uniform and three kinds of functionally graded distributions of carbon nanotubes along thickness direction of the panels. Material properties of nanocomposite panels are estimated by employing an equivalent continuum model based on the Eshelby–Mori–Tanaka approach. The governing equations are developed based on the first-order shear deformation shell theory. Detailed parametric studies have been carried out to reveal the influences of volume fraction of carbon nanotubes, edge-to-radius ratio and thickness on flexural strength and free vibration responses of the panels. In addition, effects of different boundary conditions and types of distributions of carbon nanotubes are examined.

1. Introduction

Recently, Carbon nanotubes (CNTs) have been widely accepted as a new advanced material with high strength and stiffness and a high aspect ratio and low density. Numerous investigators have reported remarkable physical and mechanical properties of this new form of carbon and CNTs may be selected as an excellent candidate for reinforcement of polymer composites. Sun et al. [1] analytically studied the axial Young's modulus of single-walled carbon nanotube arrays with diameters ranging from nanometer to meter scales. Their results confirmed that CNTs have mechanical properties superior than carbon fibers.

Researchers have analytically, experimentally and numerically investigated the constitutive models and mechanical properties of carbon nanotube polymer composites. Coleman et al. [2] reviewed and compared mechanical properties of single- and multi-walled carbon nanotube reinforced composites fabricated by various processes, in which the composites based on chemically modified nanotubes showed the best results since functionalization significantly enhances both dispersion and stress transfer. Tensile tests of CNT composites indicated that reinforcement with only 1 wt% nanotubes results in 36–42% increase in elastic modulus and 25% increase in breaking stress [3]. Odegard et al. [4] presented constitutive models of nanotubes-reinforced polymer composites with the nanotube, the local polymer near the nanotube and the nanotube/polymer interface modeled as effective continuum fibers, using an equivalent-continuum modeling method. By using molecular dynamic simulations, Griebel and Hamaekers [5] examined the elastic moduli of polymer-carbon nanotube composites with a single-walled carbon nanotube embedded in polyethylene. The results showed an excellent agreement with the macroscopic rule of mixtures. Based on the Mori–Tanaka effective-field method, Shi et al. [6] investigated effect of nanotube waviness and agglomeration on elastic properties of carbon nanotube reinforced composites.

Structure elements (beam, plate and shell) play an important role in actual structural applications. Carbon nanotube-reinforced composite (CNTRC) is an advanced material that can be embedded in beam, plate or shell as structural components. Bending behavior of one-dimensional structures is an important consideration in the design of structural components. Wuite and Adali [7] presented a multiscale analysis of deflection and stress behavior of symmetric cross-ply and angle-ply laminated CNTRC beams. Yas and Samadi [8] analysed free vibration and buckling of nanocomposite Timoshenko beams reinforced by single-walled carbon nanotubes (SWCNTs) resting on an elastic foundation using the generalized differential quadrature method. By employing an equivalent continuum model that follows the Eshelby–Mori–Tanaka approach, Formica et al. [9] studied vibration behaviors of CNTRC plates. Arani et al. [10] analytically and numerically investigated buckling behaviors of laminated composite plates in which optimal orientations of

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CNTs required to achieve the highest critical load and the corresponding mode shapes were calculated for different kinds of boundary conditions, as well as aspect ratios of the plates. Motivated by the concept of functionally graded materials, some further investigations about functionally graded carbon nanotube reinforced composites (FG-CNTRC) have been conducted. With carbon nanotubes assumed graded in thickness direction of beams, Ke et al. [11] investigated nonlinear free vibrations of functionally graded nanocomposite beams. By using the mesh-free kp-Ritz, Lei et al. [12] analysed buckling of FG-CNTRC plates under various in-plane mechanical loads. Large deformation behaviors of FG-CNTRC plates were investigated in [13]. Wang and Shen [14] studied large amplitude vibration of FG-CNTRC plates resting on an elastic foundation in thermal environments. Aragh et al. [15] studied natural frequency characteristics of a continuously graded CNT-reinforced cylindrical panel, based on the Eshelby–Mori–Tanaka approach. For FG-CNTRC cylindrical shells, Shen and Xiang [16] examined the large amplitude vibration behavior of nanocomposite cylindrical shells in thermal environments. With FG-CNTRC cylindrical shells subject to axial compression and lateral pressure, postbuckling behaviors in thermal environments were analysed in [17,18].

The present work analyses flexural strength and free vibration of functionally graded carbon nanotube reinforced composite (FG-CNTRC) cylindrical panels. The mesh-free *kp*-Ritz method based on the first-order shear deformation shell theory is employed to derive the discretized governing equations. The CNTs are assumed to be uniaxially aligned in axial direction and functionally graded in thickness direction of the panels. The effective material properties of FG-CNTRC cylindrical panels are estimated through a micromechanical model based on the Eshelby–Mori–Tanaka approach. Several computational simulation examples are presented to figure out the effects of volume fraction of CNTs, edge-to-radius ratio, thickness, boundary conditions and distribution types of CNTs on flexural strength and free vibration responses of the panels.

2. Carbon nanotube reinforced composite panels

The configuration of the cylindrical panel considered in this paper is shown in Fig. 1. This panel is assumed to be thin and of length L, radius R, span angle θ_0 and thickness h. As shown in Fig. 2, the CNTs are assumed to be uniaxially aligned in axial direction and functionally graded in thickness direction of the cylindrical panels, that is, UD is uniformly distributed; FG-V, FG-O and FG-X denote the other three types of functionally graded distributions of CNTs. For FG-V type panel, the top surface of the cylindrical panel is CNT-rich. For FG-O type panel, the middle surface of the cylindrical panel is CNT-rich and both top and bottom surfaces are CNT-rich for FG-X type panel. According to distributions of CNTs in the thickness direction of cylindrical panels, CNT volume fractions $V_{CNT}(z)$ are expressed as

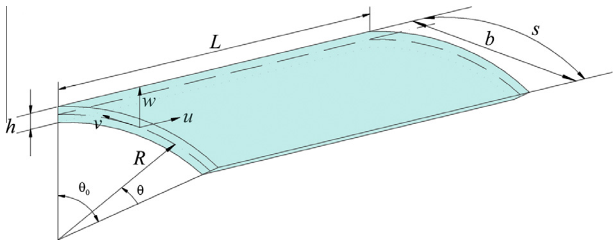

Fig. 1. Geometry properties of CNTRC panel.

$$V_{CNT}(z) = \begin{cases} V^*_{CNT} & \text{(UD)} \\ \left(1+\frac{2z}{h}\right)V^*_{CNT} & \text{(FG-V)} \\ 2\left(1-\frac{2|z|}{h}\right)V^*_{CNT} & \text{(FG-O)} \\ 2\left(\frac{2|z|}{h}\right)V^*_{CNT} & \text{(FG-X)} \end{cases}, \tag{1}$$

where

$$V^*_{CNT} = \frac{w_{CNT}}{w_{CNT} + (\rho^{CNT}/\rho^m) - (\rho^{CNT}/\rho^m)w_{CNT}}, \tag{2}$$

where w_{CNT} is the fraction of mass of the CNTs, and ρ^m and ρ^{CNT} are densities of the matrix and CNTs, respectively.

Since the effective material properties of CNT-reinforced materials are sensitive to the structure of CNTs [19–22], several micromechanical models have been proposed to predict the effective material properties of CNT-reinforced nanocomposites, such as Eshelby–Mori–Tanaka scheme [9,23,24] and the extended rule of mixture [17,25,26]. According to Benveniste's revision [27], effective elastic module tensor $\mathbf{L}$ can be expressed as

$$\mathbf{L} = \mathbf{L}_m + V_{CNT}\langle(\mathbf{L}_{CNT} - \mathbf{L}_m)\cdot\mathbf{A}\rangle\cdot[V_m\mathbf{I} + V_{CNT}\langle\mathbf{A}\rangle]^{-1}, \tag{3}$$

where $\mathbf{I}$ is the fourth-order unit tensor and $\mathbf{L}_m$ and $\mathbf{L}_{\text{CNT}}$ are stiffness tensors of the matrix and CNT, respectively. The angle brackets represent an average over all possible orientation of the inclusions. $\mathbf{A}$ is the diluted mechanical strain concentration tensor and is written as

$$\mathbf{A} = [\mathbf{I} + \mathbf{S}\cdot\mathbf{L}_m^{-1}\cdot(\mathbf{L}_{CNT} - \mathbf{L}_m)]^{-1}, \tag{4}$$

where $\mathbf{S}$ is the fourth-order Eshelby tensor [28] and is well defined for cylindrical inclusions in [29].

3. Theoretical formulations

3.1. Displacement filed and strains of CNTRC panels

According to the first-order shear deformation shell theory [30], the displacement field is expressed as

$$u(x,\theta,z) = u_0(x,\theta) + z\phi_x(x,\theta), \tag{5}$$

$$v(x,\theta,z) = v_0(x,\theta) + z\phi_\theta(x,\theta), \tag{6}$$

$$w(x,\theta,z) = w_0(x,\theta), \tag{7}$$

where $(u_0, v_0, w_0, \phi_x, \phi_y)$ are displacement components at the middle surface of the panels ($z = 0$).

The strain–displacement equations are given as

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{\theta\theta} \\ \gamma_{x\theta} \end{Bmatrix} = \boldsymbol{\varepsilon}_0 + z\boldsymbol{\kappa} = \begin{Bmatrix} \frac{\partial u_0}{\partial x} \\ \frac{1}{R}\frac{\partial v_0}{\partial \theta} + \frac{w_0}{R} \\ \frac{1}{R}\frac{\partial u_0}{\partial \theta} + \frac{\partial v_0}{\partial x} \end{Bmatrix} + z\begin{Bmatrix} \frac{\partial \phi_x}{\partial x} \\ \frac{1}{R}\frac{\partial \phi_\theta}{\partial \theta} \\ \frac{1}{R}\frac{\partial \phi_x}{\partial \theta} + \frac{\partial \phi_\theta}{\partial x} \end{Bmatrix}, \tag{8}$$

$$\begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = \boldsymbol{\gamma}_0 = \begin{Bmatrix} \phi_\theta + \frac{1}{R}\frac{\partial w_0}{\partial \theta} - \frac{v_0}{R} \\ \phi_x + \frac{\partial w_0}{\partial x} \end{Bmatrix}. \tag{9}$$

3.2. Energy functional of analysis of flexural strength and free vibration of CNTRC cylindrical panels

For analysis of flexural strength, the panels are subjected to uniform transverse pressure loading $\mathbf{q}$, the strain energy of CNTRC cylindrical panels is given as

$$U_\varepsilon = \frac{1}{2}\int_0^L \int_0^{\theta_0} \boldsymbol{\varepsilon}^T\mathbf{S}\boldsymbol{\varepsilon}R\,d\theta\,dx, \tag{10}$$

where