Composite Structures 111 (2014) 271–284

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

A unified accurate solution for vibration analysis of arbitrary functionally graded spherical shell segments with general end restraints

Zhu Su, Guoyong Jin *, Shuangxia Shi, Tiangui Ye

College of Power and Energy Engineering, Harbin Engineering University, Harbin 150001, PR China

ARTICLE INFO

Article history:
Available online 11 January 2014

Keywords:
Functionally graded spherical shell
Shell segments
Free vibration
General end restraints

ABSTRACT

A unified accurate solution procedure for free vibration analysis of arbitrary functionally graded spherical shell segments with general end restraints is presented. The material properties of the spherical shells are assumed to change continuously in the thickness direction and two different four-parameter power-law distributions are considered. The proposed method is formulated by the Ritz procedure on the basis of the first-order shear deformation shell theory. Each of admissible functions, regardless of boundary conditions, is composed of a standard Fourier cosine series and several auxiliary functions introduced to ensure and accelerate the convergence of series representations. The accuracy and reliability of the current solution are validated by comparing the results with existing results and those generated from the finite element analyses, and numerous new results for functionally graded spherical shells subjected to elastic restraints are presented, which can serve as the benchmark solutions for other computational techniques in the future research. The effects of the boundary conditions, power-law exponents, and shell segments on the free vibrations of the spherical shells are also investigated, and some interesting insights into the parameter effects on frequency behaviors are illustrated.

1. Introduction

Functionally graded materials (FGMs) are a new of composite materials, in which the material properties vary smoothly and continuously from one surface of the material to the other surface. The unique properties can eliminate the larger inter-laminar stresses which result from the abrupt change of material properties at the interface between the layers of conventional laminated composite structures. Therefore, FGMs are extensively applied in various fields. Spherical shells are important structural components in engineering applications due to their special geometric shapes. With the increased use of spherical shells made of FGMs, a thorough understanding of their vibration characteristics is essential for designers and engineers.

Extensive investigations have been conducted to analyze the free vibration of isotropic and laminated spherical shells in the past [1–19]. There are some studies regarding spherical shells which are based on the classical shell theory (CST) where the effects of shear and normal deformations in the thickness direction are neglected [1–3]. Since the CST is only valid for thin shells, many researchers analyzed free vibration of the moderately thick spherical shells on the basis of the first-order shear deformation theory (FSDT), such as Artioli et al. [4], Qu et al. [5], Lee [6], Gautham and Ganesan [7], Wu and Heyliger [8], Ferreira et al. [9], Prasad [10] and Kalnins [11]. In the FSDT, shear deformation effect is regarded and shear correction factors are introduced to adjust the transverse shear stiffness. In order to have a better prediction of the vibration behaviors for thick spherical shells, the higher-order shear deformation theory (HSDT) and three-dimensional (3-D) elasticity theory have been used by several investigators, such as Sai Ram and Sreedhar Babu [12], Viola et al. [13], Panda and Mahapatra [14], Chen et al. [15,18], Ding and Chen [16], Wu and Lo [17] and Kang and Leissa [19]. More detailed descriptions regarding the shell theories may be found in several monographs respectively by Leissa [20], Qatu [21], Reddy [22], and Carrera [23]. Apart from the aforementioned shell theories, many computational methods are applied in the vibration analysis of spherical shells, such as finite element method [3,12], pseudospectral method [6], meshless method [9], generalized differential quadrature (GDQ) method [4,13], state-space method [15], Frobenius powers series method (FPSM) [16,18], perturbation method [17] and Ritz method [19].

Some few publications [24–31] on the analysis of functionally graded spherical shells have been reported in literature. Reddy and Cheng [24] found exact correspondences for vibration frequencies of a functionally graded spherical shallow shell using different theories including the classical theory and first-order and third-order shear deformation theories. Ganapathi [25] studied the dynamic stability behavior of a clamped functionally graded material spherical shell structural element subjected to external pressure.

* Corresponding author. Tel.: +86 451 82589199.
E-mail address: guoyongjin@hrbeu.edu.cn (G. Jin).

0263-8223/$ - see front matter © 2014 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.compstruct.2014.01.006

The structural model is based on FSDT and geometric non-linearity is considered. Qu et al. [26] presented a general formulation for vibrations analysis of functionally graded shells subjected to arbitrary boundary conditions, in which the formulation is derived by means of a modified variational principle in conjunction with a multi-segment partitioning procedure on the basis of the FSDT. Free vibrations of functionally graded thin and thick shells are investigated by Neves et al. [29] using radial basis functions collocation based on HSDT. Wu and Tsai [30] presented a 3-D solution for the static analysis of functionally graded annular spherical shells by using differential quadrature method and asymptotic expansion.

From the review of the literature, the literature available on the vibration of FGM spherical shells is limited, and most of those limited literature are confined to shallow spherical shells and hemispherical shells with and without circular opening around the poles subjected to the classical boundary conditions. However, there are many kinds of shape about spherical shells segments and non-classical boundary conditions such as elastic boundaries, which are often encountered in practical engineering applications, and there is a considerable lack of corresponding research regarding those spherical shells with general boundary conditions.

In this paper, a unified accurate solution procedure for the vibration analysis of arbitrary functionally graded spherical shell segments with general boundary conditions is presented. The material properties of the spherical shells are assumed to change continuously in the thickness direction and two different four-parameter power-law distributions are considered. The proposed method is formulated by the Ritz procedure on the basis of the first-order shear deformation shell theory. Each of admissible functions, regardless of boundary conditions, is composed of a standard Fourier cosine series and several auxiliary functions introduced to ensure and accelerate the convergence of series representations. Mathematically, such a series expansion is capable of representing any function including the exact solutions. The accuracy and reliability of the current solution are validated by comparing the results with existing results and those generated from the finite element analyses, and numerous new results for functionally graded spherical shells subjected to elastic restraints are presented, which can serve as the benchmark solution for other computational techniques in the future research. The effects of the boundary conditions and material power-law distribution on the free vibration of the spherical shells are also investigated.

2. Functionally graded spherical shells

A functionally graded spherical shell segment with radius R is considered, as shown in Fig. 1. The geometry and dimensions of the shell segment are defined with respect to the coordinates φ, θ and z along the meridional, circumferential and radial directions which is located in the middle surface of the spherical shell. The spherical shell segment domain is bounded by $0 \leqslant \varphi \leqslant \varphi_1 - \varphi_0$, $0 \leqslant \theta \leqslant 2\pi$, $-h/2 \leqslant z \leqslant h/2$. The arbitrary spherical shells can be obtained by setting different values to φ_0 and φ_1. The displacement components of an arbitrary point within the spherical shell segment domain in the φ, θ and z directions are designated by $\bar{u}$, $\bar{v}$ and $\bar{w}$. The distance of each point from the axis of revolution is given by $R_1 = R\sin(\varphi + \varphi_0)$. Each edge of the spherical shell segment

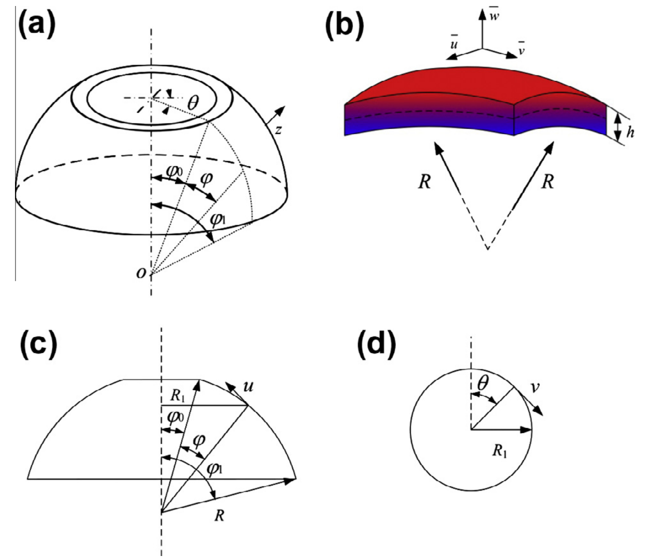

Fig. 1. The coordinate system and geometry of a spherical shell segment: (a) the coordinate system; (b) a differential element of the FGM spherical shell segment; (c) meridional section of middle surface; and (d) circumferential section of middle surface.

is restrained by three sets of independent linear springs (k_u, k_v, k_w) and two sets of rotational springs (K_φ,K_θ) to simulate the given or typical boundary conditions. The clamped boundary (C) can be simulated by assuming the springs' stiffness equal to infinity, which is represented by a very large number, 1×10^{15} N/m. and a free boundary (F) can be obtained by assuming the springs' stiffness equal to zero. The corresponding spring stiffnesses for four types of classical boundaries are given in Table 1.

Typically, the functionally graded materials are made of a mixture of ceramic and metal. The material properties are assumed to vary smoothly and continuously along the thickness direction and can be expressed as:

$$P(z) = (P_c - P_m)V_c(z) + P_m \tag{1}$$

where P represents the material properties of constituents including Young's modulus $E(z)$, density $\rho(z)$ and Poisson's ratio μ (z). The subscripts c and m represent the ceramic and metallic constituents, respectively. V_c is the volume fraction of the ceramic and follows two general four-parameter power-law distributions:

$$FGM_{I(a/b/c/p)}: \qquad V_c = \left[1 - a\left(\frac{1}{2} + \frac{z}{h}\right) + b\left(\frac{1}{2} + \frac{z}{h}\right)^c\right]^p \tag{2.a}$$

$$FGM_{II(a/b/c/p)}: \qquad V_c = \left[1 - a\left(\frac{1}{2} - \frac{z}{h}\right) + b\left(\frac{1}{2} - \frac{z}{h}\right)^c\right]^p \tag{2.b}$$

where the power-law exponent p and parameters a, b and c determine the material variation profile through the functionally graded shell segment thickness. The variations of volume fraction V_c for different values of the parameters a, b, c and p are depicted in Fig. 2. More detailed descriptions on the material variation profile of FGMs can be found in Ref. [27].

Table 1
The corresponding spring stiffnesses for the classical boundary conditions.

BC	Essential conditions	k_u	k_v	k_w	K_φ	K_θ
Clamped (C)	$u = v = w = \psi_\varphi = \psi_\theta = 0$	$1e^{15}$	$1e^{15}$	$1e^{15}$	$1e^{15}$	$1e^{15}$
Simply-supported (SS)	$u = v = w = \psi_\theta = 0$	$1e^{15}$	$1e^{15}$	$1e^{15}$	0	$1e^{15}$
Shear-diaphragm (SD)	$v = w = 0$	0	$1e^{15}$	$1e^{15}$	0	0
Free (F)	No constraints	0	0	0	0	0