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The Poly-SAFE method: A semi-analytical representation of finite element models via nested polynomial reduction of modal data

Diego Cárdenas [a], Hugo Elizalde [a], Pier Marzocca [b], Oliver Probst [c,*], Ricardo Ramirez [a], Juan P. Toledo [a]

[a] School of Design, Engineering and Architecture, Tecnológico de Monterrey, Campus Ciudad de México, Calle del Puente 222, Col. Ejidos de Huipulco, Tlalpan, CP14380 México, DF, Mexico

[b] Mechanical and Aeronautical Engineering Department, Clarkson University, CAMP 234, Potsdam, NY 13699-5725, United States

[c] Physics Department, Tecnológico de Monterrey, Campus Monterrey, Av. Eugenio Garza Sada 2501 Sur, Col. Tecnológico, Monterrey, CP64849 México, Mexico

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ABSTRACT

This paper introduces a novel semi-analytical representation of a displacement-based finite element model reduced via nested polynomials obtained through fitting of modal data. This method, termed Poly-SAFE (Polynomial Semi-Analytical Finite Element), is particularly suitable for modelling thin-walled composite structures subject to recursive analyses under varying loads, a common scenario in fluid–structure interaction (FSI) and Progressive Failure Assessment (PFA). The resulting functionals, i.e. polynomials inside polynomials, can be evaluated in an analytical fashion to yield displacements at arbitrary positions not limited to typical finite element grid nodes. These functionals remain virtually load-independent, allowing a Poly-SAFE model to be constructed without previous knowledge of magnitude, direction and location of applied loads, either static or dynamic. In this paper the theoretical framework of the Poly-SAFE method is presented in some detail, followed by an application of the theory to an extruded airfoil-shaped, laminated thin-walled beam subject to static loads. The displacement field captured by the new method is compared to the predictions of its associated finite element model, showing an excellent overall agreement. Finally, the advantages of Poly-SAFE against FE models in specific analyses and contexts are emphasised.

1. Introduction

Recent manufacturing advances and the wider availability of new materials have allowed engineers to replace heavy metallic structures with stronger, stiffer, and lighter counterparts. This has led to the construction of larger, thinner, and hollowed components fabricated from multi-layered laminates, widely known as thin-walled composite structures (TWCS) [1–4]. Examples include satellite antennas, airplane fuselages and wings, submarines, helicopter blades, and wind turbine blades [5], where the latter can be tailored to withstand very specific loading conditions taking advantage of anisotropic material constituents and non-homogeneous material distributions [6,7]. The fast pace at which empirical solutions are adopted in complex scenarios places enormous challenges on designers, who lack practical mathematical models and analysis tools (in terms of handiness and computational economy) for ensuring structural integrity with competitive safety factors [8]. As a result, TWCS are nowadays still designed with pretty much the same tools as their older metallic counterparts, leading to high safety factors and leaving much of the potential for weight reduction uncharted [9]. On the other hand, the steep rise in computational power experienced during the last decades has prompted engineers to attempt increasingly complex endeavours on personal computers. For example, fluid–structure interaction (FSI) and Progressive Failure Assessment (PFA) simulations, once restricted to the realm of specialized super-computers, are now routinely solved by off-the-shelve computational tools [10–12]. Nonetheless, real-world engineering challenges are always one step beyond the available computing power and can easily supersede those advances, as it is the case for nonlinear recursive analyses of large structural models. Such scenarios appear frequently, for example, in complex aerodynamic and aeroelastic simulations of wind turbine composite blades. A brief literature survey [13] shows that there are basically four alternatives to model the dynamic response of TWCS, depending on the purpose of the analysis.

1.1. Finite element (FE) 3D models

A detailed calculation of the displacement, strain and stress fields can only be attained via large 3D FE models containing all geometric and material features [14–16]. However, computational requirements become prohibitive when the interest is to couple FEM with CFD (Computational Fluid Dynamics) models; therefore,

* Corresponding author. Tel.: +52 8183582000.
E-mail address: oprobst@itesm.mx (O. Probst).

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cheaper alternatives such as the Blade Element Momentum (BEM) methods are typically used for calculating aerodynamic loads [17,18]. A popular choice for computational economy is to couple a BEM model with a generic beam element (based on classical Euler or Timoshenko theories, as found in most commercial software), providing a simple, inexpensive and reasonably accurate tool for predicting the aeroelastic structural response under steady aerodynamic conditions. This approach has been implemented in the aeroelastic codes Alcyone, GAST, HAWC and PHATAS [19–22]. However, it has several limitations including the fact that strain/stress fields at cross sections, required by most aeroelastic analyses, are difficult or impossible to recover, and multi-layer materials and/or thin-walled structures cannot be treated effectively.

1.2. Advanced thin-walled beam (TWB) FE models

These models condensate the full 3D geometrical and material data into equivalent beam elements with just a few Degrees of Freedom (DOF) per node [3,4,23–27]. Librescu and co-workers [4,6,27] developed a specialized TWB theory and FE discretization with coupled tensile/torsional/bending capabilities, where shell properties at given cross-sections are integrated around the axis, thereby producing an effective 1D element with 7-DOF per node and the possibility of recovering one axial and two shear stresses at the outer shell. However, a requirement of this theory is that cross-sections remain non-deformable, thus limiting its ability to treat relatively thin shells. Hodges [23] developed a geometrically exact, intrinsic theory for the dynamics of curved and twisted anisotropic beams, by partitioning the analysis into a 2D cross-sectional and 1D axial components, capable of recovering a detailed stress field map at each cross-section defined. This model was later extended to account for non-uniform, anisotropic cross-sections, thereby representing a good alternative for retrieving realistic stress fields at the outer skin. In [27–30], a parametric structural model of wings based on an exact kinematic approach was formulated and coupled with an incompressible unsteady aerodynamic model obtained via an indicial formulation accounting for viscous effects, including dynamic stall and flow separation.

1.3. Modal analysis

The size of an FE model can be drastically reduced to a linear superposition of a few representative mode-shapes, with sufficient accuracy to capture structural dynamic response [31]. When combined with generic beam elements, it provides perhaps the cheapest alternative to model TWCS, hence this approach has been implemented in most aeroelastic codes such as FAST, FLEX4, FLEX5, GAROS, GH-Bladed and VIDYN [19–22]. On the other hand, there are a number of modal techniques tailored to handle moderate geometrical non-linearities, although they are rarely found in commercial codes [35]. However, depending on the degree of truncation (a few, in general one or two, decoupled modes are retained in the model) and the type of FE model (usually generic beam elements), this method could be stripped of its ability to reproduce a representative stress field at the outer shell, which is important for design and damage assessment.

1.4. Multi-body dynamics

The TWCS is here approximated by a number of rigid elements connected by rotational springs and hinges, thus capable of simulating large deflections where the response is obtained by solving coupled multi-body dynamic equations. This procedure is implemented in the aeroelastic codes ADAMS/WT [35], DUWECS [36], FLEXLAST, HAWC2, TURBU [19–22]. This approach lies between FE and Modal Analysis in terms of computational effort and is evidently a good choice for obtaining an overall dynamic response in terms of displacements, although strain/stress fields cannot be retrieved at all.

In view of the exposition above, it can be concluded that the current approaches are either too simplified to provide detailed strain/stress fields or too complex to allow for recursive coupled analyses such as fluid–structure interaction or damage propagation with reasonable computational resources and execution times. Consequently, there exists a gap for a detailed but inexpensive model which allows performing recursive (i.e. iterative or nonlinear) analyses while still yielding results with an accuracy comparable to that obtained with industry-standard 3D FE models. To deal effectively with varying-load scenarios, as often the case in FSI, PFA, and similar analyses, load-independent models are also preferable. This paper presents a novel methodology for the analysis of TWCS, here called the *Poly-SAFE* (Polynomial Semi-Analytical Finite Element) method, proposed to address shortcomings of other approaches and satisfy the aforementioned requirements. This model is based on nested polynomial reduction of modal data generated by a finite element model, yielding a semi-analytical expression for the displacement field, which can be evaluated at any physical coordinate, not just at FE nodes. This semi-analytical expression remains virtually load-independent and is therefore particularly suited for scenarios anticipating multiple re-analyses under variable loads (both deterministic and stochastic) and/or frequent localization of critical points. Under such conditions, Poly-SAFE is expected to offer a significant computational advantage of several orders of magnitude as compared to its associated FE model.

The scope of this paper includes a description of the theoretical framework of Poly-SAFE as well as its validation in the case of a TWCS sample structure subject to static loads, where the displacement field obtained by the Poly-SAFE model is compared to the predictions of its parent FE model. A validation of the Poly-SAFE approach for dynamic loads will be presented in a follow-up paper, along with the derivation of similar semi-analytical expressions for the strain and stress fields. The rest of the paper is organized as follows: Section 2 opens with a summary of key concepts of modal analysis as a basis for further developments. Then, Section 3 proceeds with a detailed theoretical and practical description of the Poly-SAFE method; the description is illustrated with results obtained with a Poly-SAFE model of a sample TWCS. Section 4 describes the object of a study based on an airfoil-shaped, laminated thin-walled beam (previously reported in [37–40]) and provides a definition of the load cases studied. In Section 5, results of the modelling of the displacement field for each of the load cases are presented and the Poly-SAFE results are compared to those obtained with its parent FE model. Finally, Section 6 summarizes the findings and provides an outlook for future work.

2. Brief background on modal analysis

A standard FE model of a structural system can be represented as [31]:

$$\mathbf{m}_{n\times n}\ddot{\mathbf{u}}(t)_n + \mathbf{c}_{n\times n}\dot{\mathbf{u}}(t)_n + \mathbf{k}_{n\times n}\mathbf{u}(t)_n = \mathbf{F}(t)_n \quad (1)$$

where $\mathbf{m}$, $\mathbf{c}$, $\mathbf{k}$ are $n \times n$ mass, damping, and stiffness physical matrices, respectively, and n is the system size in terms of Degrees of Freedom (DOFs); $\mathbf{F}(t)$ and $\mathbf{u}(t)$ are time-dependent vectors of externally applied loads and physical displacements, respectively, both of size n. For continuous systems, matrices in Eq. (1) are usually full, thus direct solution is cumbersome, and a modal approach is preferred. To this end, the physical displacement $\mathbf{u}(t)$ is expressed as a linear superposition of generalized (i.e. modal) displacements as follows: