Composite Structures 111 (2014) 354–361

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

# An analytical model for the secondary bending prediction in single-lap composite bolted-joints

A. Olmedo, C. Santiuste *, E. Barbero

Department of Continuum Mechanic and Structural Analysis, University Carlos III of Madrid, Avda de la Universidad 30, 28911 Leganés, Madrid, Spain

ARTICLE INFO

*Article history:*
Available online 27 January 2014

*Keywords:*
Composite
Bolted joints
Analytical model
Secondary bending
Aeronautical structures

ABSTRACT

The development of an enhanced analytical approach to predict the secondary bending of single-lap composite bolted joints is presented. The enhanced accounts for the estimation of secondary bending as a function of geometrical parameters, material properties, and stacking sequence. The model is validated through comparison of the predicted load–displacement curve with experimental tests and literature data. The model accuracy was validated for different values of bolt torque, friction coefficient, geometrical parameters, and material properties. The method has been also used in a parametric study to analyse the influence of the main joint parameters on the load–displacement curve. The model is a valuable preliminary design tool for analysing the influence of the joint parameters on the stiffness of single-lap composite joints including the effect of secondary bending.



## 1. Introduction

Composite materials combine fatigue and corrosion resistance, light weight and high specific stiffness and strength. These properties make composites suitable for a wide range of high responsibility applications in aeronautic industry. Mechanical joining is the most important method of assembling composite structural elements in the aerospace industry, due to its facility to assemble, disassemble and repair, and its tolerance to environmental effects [1]. However, fastener joints should be carefully designed due to the stress concentration at the surrounding of the hole. The stress concentration in composite laminates can be more critical than in metallic components [2,3]. The study of bolted joints in composite structural elements has received considerable attention in both the scientific literature and aeronautical standards [4–9].

The bolted joint performance depends on different parameters, mainly: joint geometry, plates and bolt materials, laminate layup, clearance, friction between different elements of the joint, temperature, load path, and bolt torque. The complex mechanical behaviour of composite bolted joints has been investigated by several researches by means of experimental studies, see for example [10–12]. However, due to the large range of parameters, the use of purely experimental techniques would be prohibitively expensive. The development of theoretical reliable models is necessary to get a better understanding of all the aspect of the joint behaviour and to optimise the design of composite bolted joints.

Due to the complexity of composite bolted joints, several authors have proposed numerical models to analyse its mechanical behaviour but few studies have focused on the development of analytical models. The stiffness and the strength of bolted joints have been predicted with finite element models, showing good agreement with experimental data, [13–16]. Despite the accuracy of the finite element method to predict the failure of composite elements under different loading conditions [17,18], the development of simplify models can lead to a better understanding of the mechanical behaviour of composite bolted joints. Analytical models comprise the ability to explicitly describe the physical behaviour of bolted joints, and the possibility for conducting parametric studies.

The analyses of single-lap bolted joints have shown a non-uniform stress distribution throughout the thickness of composite plates in the surroundings of the hole due to the secondary bending produced by the eccentric load path [19–21]. This complex effect has been included in the formulation of simplified analytical models by mean of experimental coefficients [22–26].

Tate and Rosenfeld introduced a mass-spring model for double-lap joints made from isotropic materials [22]. Rosenfeld continued this work including experimental tests to validate the analytical model [23]. This simplified model was modified by Nelson et al. [24] to be applied to single-lap joints with anisotropic materials. To consider the influence of secondary bending on the joint stiffness, an experimental coefficient depending on the bolt

* Corresponding author. Tel.: +34 916249920; fax: +34 916249973.
*E-mail address:* csantius@ing.uc3m.es (C. Santiuste).

http://dx.doi.org/10.1016/j.compstruct.2014.01.015
0263-8223/

configuration was included in the formulation. McCarthy et al. [25] modified the mass-spring model to add the effect of clearance. They predicted accurately the load–displacement curve and the load distribution obtained in experimental tests on single-column multi-bolt composite joints. McCarthy and Gray [26] improved the mass-spring model considering two steps in the load–displacement curve: the first dominated by friction forces and the second controlled by the contact stresses. The load–displacement curve was in agreement with experimental data. All these works considered an experimental coefficient to include the effect of secondary bending in the joint stiffness.

However, the results were validated with experimental data obtained from tests on single-column multi-bolt joints where the effect of secondary bending is limited. For an accurate prediction of the load–displacement curve in single-lap single-bolted joint configurations, secondary bending effect must be predicted with a more accurate method.

In this work, an analytical model to predict the stiffness of single-lap composite bolted joint is proposed. The model includes a predictive method to consider the effect of secondary bending as a function of geometrical parameters, material elastic properties, stacking sequence, and load path eccentricity. Experimental tests were developed to validate the accuracy of the model results for different bolt torques. Also experimental data from literature were used to validate the analytical model analysing other materials and geometries. Moreover, a parametric study is include to investigate the influence of torque, friction coefficient, clearance, joint geometry, thickness, and material properties on the stiffness of composite bolted-joints.

## 2. Model description

The present analytical model is an extension of the model proposed by McCarthy and Gray [26]. Fig. 1 shows the configuration of a single-lap single-bolt composite joint. To illustrate the mechanical behaviour of a highly torqued composite joint, Fig. 2 shows a typical load–displacement curve obtained in an experimental test. The curve can be divided in three regions. The first region corresponds to a quasi-linear behaviour produced by the load being reacted solely by static friction forces acting at the shear plane. With increasing loads, the static friction forces are overcome and the laminates slip relatively to each other with a uniform load. During this second region, the bolt-hole clearance is taken up and the bolt shank begins to contact the laminates. The third region starts when significant contact is established between bolt shank and hole surface, the bolt begins to transmit loads to the laminate and the joint stiffness increases significantly leading to a new quasi-linear region. The joint stiffness in the third region is strongly influenced by secondary bending phenomenon. The third region ends when the load reaches the bearing strength of the laminate, the objective of this work is to analyse the composite joint stiffness for loads below the bearing strength. This goal means an important contribution to understand the mechanical behaviour of composites joints and a necessary previous step to the development of future models including the prediction of bearing strength in single-lap composite joints.

The single-lap single-bolt composite joint may be represented by a system of masses and springs, as shown in Fig. 1. The joint load is applied at mass 3 and reacted at the clamped end of the joint. The stiffness of composite plate 1 under tensile loads is represented by $K_{pl1}$, and composite plate 2 by $K_{pl2}$. During the first region, the joint stiffness is dominated by the top branch, where $K_{sh\text{-}pl1}$ and $K_{sh\text{-}pl2}$ are the stiffnesses under shear load of composite plates 1 and 2 respectively. When the value of the load at top branch reaches the maximum value of the friction forces that the joint can transmit, a relative displacement between laminates is produced without increasing loads until the clearance is taken up. This phenomenon is represented by a friction element, $F_{fricc}$. When the contact between bolt shank and laminates is established, the joint stiffness is controlled by the bottom branch, where $K_{bolt}$ includes the shear and bending stiffness of the bolt, the bearing stiffness of composite plates, and the secondary bending effect.

Considering that masses are free to move in the $x$-direction only, the system shown in Fig. 1 leads to a system of linear equations of the form:

$$[M]\{\ddot{x}\} + [K]\{x\} = \{F\} \quad (1)$$

For quasi-static loading, the accelerations can be neglected, leading to the next equation:

$$[K]\{x\} = \{F\} \quad (2)$$

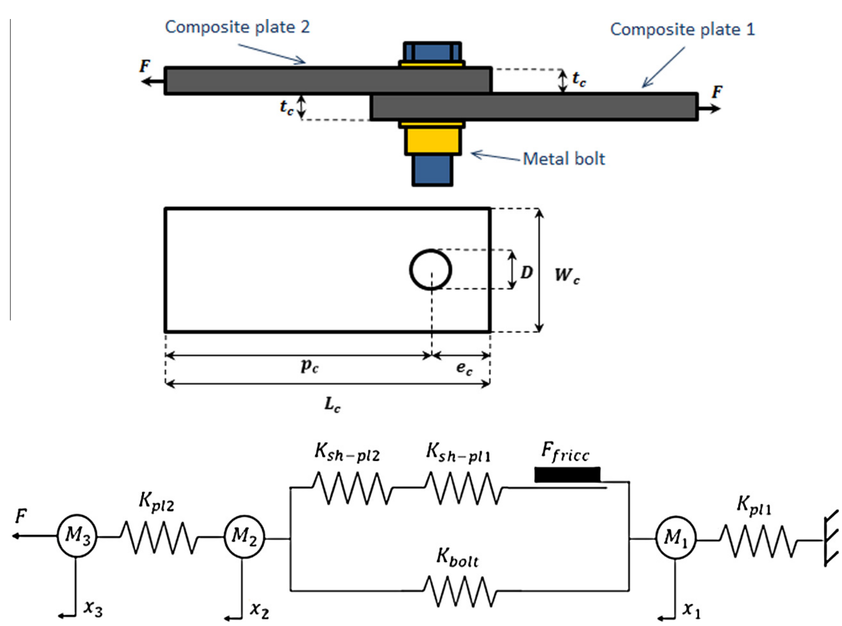


**Fig. 1.** Geometry and mass-spring model of a single-lap composite bolted-joint.