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Three-dimensional free vibration of carbon nanotube-reinforced composite plates with various boundary conditions using Ritz method

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ABSTRACT

In this paper, three-dimensional free vibration of carbon nanotube (CNT) reinforced composite rectangular plates with various boundary conditions is analyzed by developing a set of orthogonal admissible functions used in Ritz method. To validate the present analysis, numerical results were compared with those obtained in the open literature. Furthermore, the effect of CNT volume fraction, two cases of CNT distribution, boundary conditions and aspect ratio of plate on non-dimensional natural frequencies were studied. Modified rule of mixture is used to find material constants of the reinforced composite. This benchmark solution can be used to assess the accuracy of conventional two-dimensional theory.

1. Introduction

Since the discovery of carbon nanotubes [1], their plausible applications have been studied extensively, notably as reinforcing composites. Researches have been conducted to model or measure the mechanical properties of CNT or carbon nanotube reinforced composites (CNTRC). Structural mechanics approach was developed to model the molecular structures of carbon by equivalent mechanical truss and continuum structure models [2]. This idea was extended to mechanical modeling of CNTRCs [3]. This approach was repeated [4] and continued by modeling mechanical properties of CNT-polymer interphase [5]. Mori–Tanaka method was originally developed for modeling micro-scale particles in composites [6]. Formulation of this method was then rewritten in another form [7] and was presented in a final applicable form for mechanical properties of micro-composites with different fiber orientations [8]. Although Mori–Tanaka approach was originally developed for micro-composites, it was used directly to calculate material properties of laminated nanocomposite beam [9]. This approach was also used in 3D analysis of nanotube-reinforced cylindrical panel and rectangular plate [10,11]. Lie et al. [12] used Kp-Ritz method to investigate free vibration of functionally graded carbon nanotube-reinforced composite plates in thermal environment. Molecular dynamics is another approach being used directly to analyze CNTs and CNTRCs [13–16]. In the study of functionally graded ceramic–metal beams, remarkable agreement was reported between the Mori–Tanaka scheme and rule of mixture [17]. The modified rule of mixture was used in series of works on nonlinear analyses of CNTRC rectangular plates and cylindrical shells [18–21]. Computing effective material properties from Mori–Tanaka or self-consistent schemes, three-dimensional exact solution for the vibration of simply-supported functionally-graded rectangular plates was derived [22]. Three-dimensional solution of functionally graded simply supported CNTRC plate with thin piezoelectric surface layers was presented by using theory of elasticity [23]. To the authors' knowledge, however, three-dimensional free vibration analysis of FG-CNTRC rectangular plates with various edges boundary condition has not been reported yet. In the present work we attempt to carry out the aforementioned analysis. In this approach, orthogonal polynomials are developed from Jacobi polynomials to be used in Ritz analysis.

2. Problem formulation

Consider a CNTRC rectangular plate with length a, width b, and thickness h, as shown in Fig. 1. Maximum strain and kinetic energy of the vibrating plate can be written as follows

$$SE_{max} = \frac{1}{2}\int_{-\frac{h}{2}}^{\frac{h}{2}}\int_{-\frac{b}{2}}^{\frac{b}{2}}\int_{-\frac{a}{2}}^{\frac{a}{2}}[\sigma_{xx}\varepsilon_{xx} + \sigma_{yy}\varepsilon_{yy} + \sigma_{zz}\varepsilon_{zz} + 2(\tau_{yz}\varepsilon_{yz} + \tau_{xz}\varepsilon_{xz} + \tau_{xy}\varepsilon_{xy})]dxdydz \quad (1)$$

$$KE_{max} = \frac{\omega^2}{2}\int_{-\frac{h}{2}}^{\frac{h}{2}}\int_{-\frac{b}{2}}^{\frac{b}{2}}\int_{-\frac{a}{2}}^{\frac{a}{2}}\rho[u^2 + v^2 + w^2]dxdydz. \quad (2)$$

where ε_{ij} and σ_{ij} are strain and stress tensor components, respectively, while ρ is the density of composite plate. Strain–displacement relations in the rectangular Cartesian coordinates are given by

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Nomenclature

u, v, w	displacement component in x-, y-, and z-direction
a, b, c	length, width, and thickness of plate
ε_{ij}	strain components
ρ	density of CNTRC in any point
ω	natural frequency of plate
C_{ij}	elastic stiffness coefficient
V_m	matrix volume fraction in any point
V_{CN}	CNT volume fraction in any point
E_m	Young's moduli of polymer matrix
G_{ij}	shear modulus of CNTRC in ij plane
G_m	shear modulus of polymer matrix
ν_{ij}	Poisson's ratio of CNTRC in ij plane
σ_{ij}, τ_{ij}	stress components
ρ_m	density of polymer matrix
$SE_{\max}$	maximum strain energy of plate in cycle
$\mathbf{K}$	total non-dimensional stiffness matrix of plate
Π	maximum energy functional
η_1, η_2, η_3	CNT efficiency parameters
$p_i(x_j)$	ith developed polynomial
A^u_{ijk}, A^v_{lmq}, A^w_{rst}	coefficients of approximating expansion of displacements
U, V, W	non-dimensional displacement component in x-, y-, and z-direction
X, Y, Z	non-dimensional coordinates in x-, y-, and z-directions
ε^*_{ij}	strain components (revised form)
ρ^*	non-dimensional density of CNTRC
λ	non-dimensional natural frequency of plate
C^*_{ij}	non-dimensional elastic stiffness coefficient
$\overline{V}_m$	average volume fraction of matrix in plate
$\overline{V}_{CN}$	average volume fraction of CNTs in plate
E_i	Young's moduli of composite in i-direction
E^{CN}_i	Young's modulus of CNT in i-direction
G^{CN}_{ij}	shear modulus of CNT in ij plane
ν_m	Poisson's ratio of polymer matrix
ν^{CN}_{ij}	Poisson's ratio of CNT in ij plane
ρ_{CN}	density of CNT
$KE_{\max}$	maximum kinetic energy of plate in cycle
$\mathbf{M}$	total non-dimensional mass matrix of plate
$f(Z)$	distribution function of CNTs in CNTRC
$\overline{m}_{cn}$	average mass fraction of CNTs in plate
x_j	jth non-dimensional coordinate
$j^{(j,k)}_i$	ith Jacobi polynomial of order j, k

$$\varepsilon_{xx}=\frac{\partial u}{\partial x}\quad \varepsilon_{yy}=\frac{\partial v}{\partial y}\quad \varepsilon_{zz}=\frac{\partial w}{\partial z}$$
$$\varepsilon_{yz}=\frac{1}{2}\left(\frac{\partial w}{\partial y}+\frac{\partial v}{\partial z}\right)\quad \varepsilon_{xz}=\frac{1}{2}\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)\quad \varepsilon_{xy}=\frac{1}{2}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right) \tag{3}$$

where u, v, and w are displacement components in x-, y-, and z-directions, respectively. Constitutive equations of an orthotropic plate are

$$\begin{Bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{zz}\\ \tau_{yz}\\ \tau_{xz}\\ \tau_{xy}\end{Bmatrix}=\begin{bmatrix}C_{11}&C_{12}&C_{13}&0&0&0\\ C_{21}&C_{22}&C_{23}&0&0&0\\ C_{31}&C_{32}&C_{33}&0&0&0\\ 0&0&0&C_{44}&0&0\\ 0&0&0&0&C_{55}&0\\ 0&0&0&0&0&C_{66}\end{bmatrix}\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \varepsilon_{zz}\\ 2\varepsilon_{yz}\\ 2\varepsilon_{xz}\\ 2\varepsilon_{xy}\end{Bmatrix} \tag{4}$$

where C_{ij} are elastic stiffness coefficient. Substituting Eq. (4) into Eq. (1) yields

$$\begin{aligned}SE_{max}=&\frac{1}{2}\int_{-\frac{h}{2}}^{\frac{h}{2}}\int_{-\frac{b}{2}}^{\frac{b}{2}}\int_{-\frac{a}{2}}^{\frac{a}{2}}\Big[C_{11}\varepsilon_{xx}^2+C_{22}\varepsilon_{yy}^2+C_{33}\varepsilon_{zz}^2\\ &+2(C_{12}\varepsilon_{xx}\varepsilon_{yy}+C_{13}\varepsilon_{xx}\varepsilon_{zz}+C_{23}\varepsilon_{zz}\varepsilon_{yy})\Big]dxdydz\\ &+\frac{1}{2}\int_{-\frac{h}{2}}^{\frac{h}{2}}\int_{-\frac{b}{2}}^{\frac{b}{2}}\int_{-\frac{a}{2}}^{\frac{a}{2}}4\left(C_{44}\varepsilon_{yz}^2+C_{55}\varepsilon_{xz}^2+C_{66}\varepsilon_{xy}^2\right)dxdydz\end{aligned} \tag{5}$$

For simplicity, following non-dimensional physical quantities are introduced:

$$X=\frac{2x}{a}\quad Y=\frac{2y}{b}\quad Z=\frac{2z}{h}\quad U=\frac{u}{h}\quad V=\frac{v}{h}\quad W=\frac{w}{h}\quad \varepsilon^*_{ij}=\frac{ab}{h^2}\varepsilon_{ij} \tag{6}$$

From Eqs. (3) and (6), strain–displacement relations can be written as

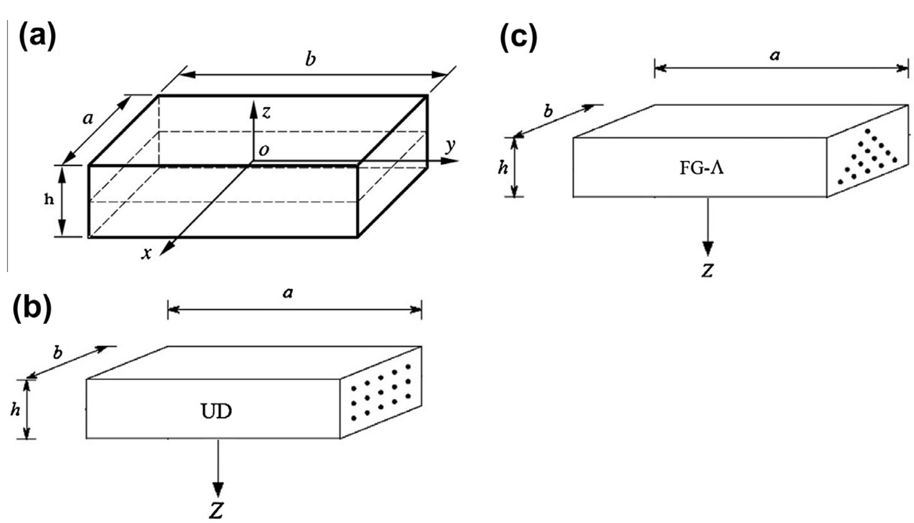

Fig. 1. Geometry and coordinate system for the CNTRC rectangular plate.