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Cohesive debonding of a stiffener from an elastic substrate

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ABSTRACT

To strengthen concrete or masonry, a modern technique uses adherent strips made of Fiber Reinforced Polymer (FRP). The model problem of an elastic stiffener pulled at one end, in adhesive contact with an elastic half plane in generalized plane stress, is here considered. An analytical solution is found under the hypothesis à *là* Baranblatt that *cohesive* adhesion forces remain active between the two materials when relative slip occurs, provided this is less than a critical limit. The stress singularity predicted by the theory of elasticity for perfect bonding is removed and the *effective bond length*, i.e., the bond length beyond which no further increase of strength is possible, coincides with the maximal length of the cohesive zone, attained when the critical slip limit is reached. The debonding process predicted by this model is in better agreement with experimental results than the predictions by other models, which neglect the deformation of the substrate.

1. Introduction and practice

Motivation for this work is a widely-used technique that employs strips/plates of Fiber Reinforced Polymer (FRP) to strengthen concrete or masonry structures. The performance of the bond between stringers and substrate plays a key role in the effectiveness of the reinforcement, so that a wide research has been focused on the mechanical response of the bonded joints in the load transfer process. Experimental results have provided a wealth of evidence that both in flexural and shear strengthening applications the most frequent failure mode is the debonding of the FRP plate from the substrate, triggered by high stress concentrations at the ends of the stiffener. Shear (mode II) crack propagation along the FRP-concrete interface eventually leads to the complete separation of the materials, causing the sudden decrease of both structural stiffness and strength.

The experimentally-observed phenomenon is of the type schematized in Fig. 1. If the axial stiffness of the stringer is high and the bond is strong,[1] the application of an axial pull-out load produces the initiation of cracking from the loaded edge; the crack slightly dives into the substrate and then propagates almost parallel to the interface a few millimeters beneath it, reaching a steady state phase of mode II propagation [1]. In fact, the maximal energy release rate is when the stringer itself is released. More precisely, a thin layer of the underlying substrate remains attached to the reinforcing stringer, but this layer is so thin that its contribution to the tension stiffening of the stringer is usually neglected: indeed, if this was not the case, the energy release associated with the stringer would be diminished. On the other hand, the contribution due to the glue layer can certainly be neglected due to its infinitesimal thickness. Therefore, a model-problem may consider the pure separation in mode II of the stringer from the substrate. In a more sophisticated analysis, it would be certainly possible to take into account the tension stiffening of the stringer, calculating its equivalent axial stiffness through an elementary homogenization procedure for the stringer itself and the substrate-layer that remains attached to it. This will not be done here but, if it was done, the analytical approach to the contact problem here presented would not change.

For the steady state of crack propagation, the model problem is the debonding in mode II of a straight[2] elastic stiffener, of prescribed length, from an elastic substrate in generalized plane stress. Since its thickness is in general very small, the FRP strip can be modeled as a membrane with negligible bending stiffness.[3] Therefore, the stiffener

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[1] In the case of FRP-to-concrete or FRP-to-masonry applications, the elastic modulus of the stringer is one order of magnitude greater than that of the substrate. The stringer is thin by wide, so that its cross sectional area is large enough to render its axial stiffness comparable with the springing constraint offered by the substrate. In general, high performance glues are used, providing a very high adhesion strength.

[2] Indeed, this model may also apply when the substrate is moderately curved [2].

[3] Freund and Suresh [3] have given a qualitative indication for the thickness of the stiffener, which has to be at least 20 times smaller than its other dimensions to be defined as a “thin” stiffener. Moreover, when the thickness is much smaller than that of the substrate (typically by a factor of 50 or more), the stringer is a “mechanically thin film”.

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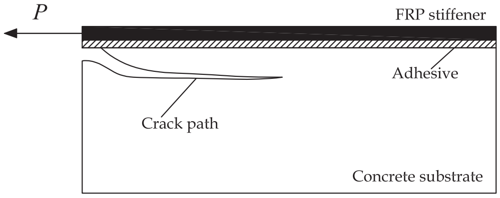

Fig. 1. Crack propagation in the pull out of a stiffener from a brittle or quasi-brittle substrate.

is not able to sustain transverse loads during small deformations and this results in the absence of peeling stresses at the interface.

Although a mixed-mode analysis [4–8] accounting for the normal stresses acting at the interface is certainly the most accurate approach, Shield and Kim [5] concluded that the *membrane* assumption can be adopted when the elasticity modulus of the stiffener is large compared to that of the substrate, while the effect of the peeling stresses must be taken into account in the case of more compliant films. From an analytical point of view, the bending stiffness may be considered through an expansion of the elastic solution with respect to the stringer thickness, where the first term represents the membrane contribution [5]. In general, bending effects are more important near the ends of the stringer: since the length-scale of the expansion must be compared with the geometry of the problem, the order of the expansion determines how close to the edges the model will yield accurate predictions. The beam theory represents the second term in the expansion, and provides significant contributions only over distances from the stringer ends of the order of its thickness [5]; the higher the elastic modulus of the stringer, the less such contributions are important. For the case at hand, considering the relative stiffness between stringer and substrate, little errors may possibly be encountered in very small neighborhoods of the stringer extremities but, apart from this, the membrane assumption agrees very well with the beam model, bringing simplifications that allow to solve the problem analytically.

Experimental tests have been conducted with different setups, including single shear tests [9–13], double shear tests [14,15] and modified beam tests [16], for which an extensive list of references can be found in Yao et al. [13] and Chen and Teng [17]. In general, in pull-out tests the axial force in the stiffener is gradually transmitted to the substrate by shear forces acting at the interface. Such forces decay very quickly passing from the loaded end to the free end of the stiffener, so that they can be considered active on a certain length only, usually referred to as the *effective bond length* or the *effective stress transfer length*. In long stiffeners, as the load increases, debonding near the applied load shifts the stress transfer zone to new areas farther away from the loading point, confirming that only part of the bond is active. In other words, the anchorage strength does not increase with an increase of the bond length beyond its active limit. However, a longer bond length may improve the ductility of the failure process due to the gradual translation of the effective length, as debonding proceeds. This phenomenon has been confirmed by many studies on steel-to-concrete [9] and FRP-to-concrete bonded joints [14].

Various shear-anchorage-strength models have been proposed to interpret this mechanism, for which a review can be found in Chen and Teng [17]. In general, these models can be classified into three categories: (i) empirical models based on the regression of test results [14]; (ii) engineering formulations based upon simplified assumptions and appropriate safety factors [15,17,18]; and (iii) fracture-mechanics-based models [19–21]. Despite the variety of the reinforcing materials, of the strengths of the substrates and of the geometry of the stiffeners, there is a general agreement on many aspects of the failure process. Since it has been experimentally verified that increasing the bond length beyond a certain limit does not lead to any increase of load-carrying capacity, all models aim at defining such limit, which coincides with the *effective bond length*.

To our knowledge, the totality of the *analytical* anchorage-strength models neglects the elastic deformation of the substrate and assumes a shear vs. slip interface constitutive law to describe the entire phenomenon. Whatever the length of the stiffener is, such models predict a fast (usually *exponential*) decay of the transfer shear stress from the loaded- to the free-end that never reaches the zero value. Since no part of the stiffener is inactive regardless of its length, the definition itself of *effective* bond length needs an engineering interpretation. For example, many researchers define the effective bond length as the bond length over which the shear bond stresses offer a total resistance which is at least 97% of the ultimate load[4] of an *infinite* joint [20,22–24]. According to other authors, the evaluation cannot but be purely experimental. Measuring the strain profile in the stiffener - usually employing resistance strain gages - the effective bond length is the length over which the strain decays from the maximum to the zero value [25,10,26–29].

There are some intrinsic ambiguities in these definitions. In the first case, there is an *a priori*-defined percentage of load and the result strongly depends upon the particular bond–slip constitutive law that is used for the model. The second definition cannot get rid of the experimental approximations and depends upon the sensitivity of the gages. In any case, all definitions implicitly assume that the deformation of the substrate is negligible, because the relative displacement between stiffener and substrate is evaluated by simply integrating the axial strain of the stiffener. The hypothesis of rigid substrate is indeed supported by the greatest majority of authors (see also Carrara et al. [12], Ferracuti et al. [30], Mazzotti et al. [11]) because it gives drastic simplifications, but it has major drawbacks, such as the implication that the slip is always nonzero whatever the bond length is.

The present article considers specifically the effect of the substrate elasticity. The resulting contact problem in plane linear elasticity is of the kind studied by other authors [31–36], with the main purpose of evaluating the stress concentrations near the edges of the stiffener in relation with crack initiation and propagation in the substrate or along the interface. More recent studies include the case of a rigid line inclusion embedded in an infinite prestressed substrate [37], to which a generic perturbation field is superimposed, as well as the case of reinforced no-tension-materials [38].

Following this rationale, in previous work [39] the authors have considered the case of a perfectly-adherent stiffener, focusing the attention on the debonding process assumed to begin, and continue, as soon as the energy release rate due to an infinitesimal delamination becomes equal to the interfacial fracture energy (Griffith balance). The main drawback of this approach was the difficulty to give a consistent definition of the *effective* anchorage length. In fact, when slip is not contemplated, the presence of the stress singularities at both ends of the stiffener produces a very rapid decay of the shear stress profile at the interface, which does not agree with experiments.

This work aims at solving this inconsistency by introducing a cohesive zone where slippage can occur. Following the approach originally proposed by Barenblatt [40], also pursued by other authors [41,5] for similar-in-type problems, the length of the cohesive zone for a fixed load is evaluated by imposing that the stress intensity factor at the end of the bonded zone is null, eliminating the singularities which are predicted by the theory of elasticity. Effective material separation is supposed to start when the relative slip exceeds a certain threshold. If the stiffener is sufficiently long,

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[4] Notice that $\tanh 2 \simeq 0.97$: this is a characteristic value in the solution of the differential equations governing the debonding process [20]. Therefore, the limit of 97% seems to be motivated by the analytical approach to the problem, rather than by sound physical considerations.