Composite Structures 111 (2014) 459–467

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

# Analytical study of the low-velocity impact response of composite sandwich beams

Inés Ivañez [1,*], Enrique Barbero [1], Sonia Sanchez-Saez [1]

*Department of Continuum Mechanics and Structural Analysis, University Carlos III of Madrid, Avda. Universidad, 30, 28911 Leganes, Spain*

ARTICLE INFO

*Article history:*
Available online 31 January 2014

*Keywords:*
Beams
Sandwich structures
Analytical modelling
Impact dynamics
Dimensional analysis

ABSTRACT

In this work the low-velocity impact response of composite sandwich beams was studied by an analytical model. A dimensional analysis was carried out in order to identify the key parameters that influence the dynamic beam response, and to assess the effect of the dimensionless groups on the contact force and contact time. Low-velocity impact tests were conducted to validate the theoretical model. The predicted results were in good agreement with experimental data in terms of maximum contact force, contact time, and contact force–time curves. It was shown that the groups with more influence on maximum contact force and contact time are the dimensionless global stiffness, the dimensionless local stiffness, and the dimensionless impact velocity.



## 1. Introduction

Composite sandwich structures are popular as primary structures in high-performance applications where minimum weight is essential and have widespread use in aerospace, automotive, and civil engineering industries. A reliable structural design must take into account the loads which can occur during its service life, and one area of concern is related to low-velocity impacts (i.e. tools falling during manufacturing and/or maintenance operations). Low-velocity impacts are considered potentially dangerous for a composite sandwich structure as the resulting damage is difficult to determine, especially when the face-sheets are made of carbon-fibre reinforced composite. Non or barely visible damage of a composite sandwich structure may be accompanied by substantial reduction of residual strength and stiffness [1,2]; therefore, it is needed to understand the effect of such impacts on their structural performance.

Three main approaches are used to analyse the impact response of composite sandwich structures: experimental testing, numerical simulations, and analytical models. Experimental studies have been conducted to describe the dynamic response of composite sandwich structures and investigate the impact damage produced by low-velocity impacts [3–5]; nevertheless, the amount of information obtained from experimental testing is limited and a broad testing programme has to be undertaken to set up an accurate experimental response. Detailed finite-element models have proved to consistently predict the impact response of sandwich structures [6–9]; however, complex numerical simulations require more computational and modelling effort. A first stage to understand the effect of impacts on structures is to build a basic model for predicting the contact force history and the overall response of the impacted structure [10]. On this point, analytical simplified models lead to more efficient tools, as they can assess global variables rapidly.

Many simplified models proposed in the literature consider the balance of energy of the system [11–13]; however, the energy-balance models simplify the dynamic of structures by assuming a quasi-static behaviour at its maximum deflection. Simplified mass-springs models take into account the dynamic of the structure and present some advantages, as they rely on measurable global variables and their predictions are easier to validate.

While there are several mass-springs models for representing the response of sandwich plates subjected to low-velocity impact by hemispherical impactors [14–17], less attention has been paid to cylindrical impactors and composite sandwich beams [18]. However, many structures can be modelled as beams (i.e. windmills blades). In addition, although mass-springs models are applied to reproduce the dynamics of sandwich structures, no systematic study on the influence of the parameters that control the low-velocity process has been found. One possible approach to this kind of study is to express the equations of the model in a non-dimensional form [19].

* Corresponding author. Tel.: +34 916249162; fax: +34 916248331.
*E-mail address:* idel@ing.uc3m.es (I. Ivañez).

[1] Mechanics of Advanced Materials Research Group, web page: http://www.uc3m.es/mma.

http://dx.doi.org/10.1016/j.compstruct.2014.01.028
0263-8223/

**Nomenclature**

| | |
|---|---|
| $A_{ij}$ | in-plane stiffness matrix of the laminate |
| $b$ | width of the beam |
| $D$ | plastic strain energy in crushing the core |
| $E_c$ | kinetic energy |
| $E_1$ | longitudinal young's modulus |
| $E_2$ | transversal young's modulus |
| $\langle E{\cdot}I\rangle$ | equivalent bending stiffness of the sandwich beam |
| $F(t)$ | contact force |
| $G_c$ | shear modulus of the sandwich core |
| $G_f$ | face-sheet shear modulus. |
| $\langle G{\cdot}A\rangle$ | equivalent shear stiffness of the sandwich beam |
| $H$ | total thickness of the beam |
| $h_c$ | thickness of the core |
| $h_f$ | thickness of the composite face-sheets |
| $K_g$ | global stiffness of the sandwich beam |
| $K_l$ | local stiffness of the sandwich beam |
| $L$ | span between supports |
| $M_0$ | impactor mass |
| $m_f$ | effective mass of the upper face-sheet |
| $m_s$ | effective mass of the sandwich beam |
| $M_S$ | total mass of the sandwich beam |
| $P$ | indentation force |
| $P_{ch}$ | characteristic force |
| $P(\delta)$ | non-linear relationship between face-sheet indentation and local displacement |
| $q$ | static core crushing strength |
| $q_d$ | dynamic core crushing strength |
| $Q_d$ | dynamic core crushing load |
| $R$ | impactor radius |
| $R_{eq}$ | effective impactor radius |
| $t$ | time |
| $t_{ch}$ | characteristic time |
| $U$ | elastic strain energy |
| $u(x)$ | local displacement field of the upper face-sheet |
| $V$ | work done by external forces |
| $V_o$ | impact velocity |
| $\delta$ | local displacement of the upper face-sheet in the contact area |
| $\dot{\delta}$ | upper face-sheet velocity |
| $\Delta$ | global displacement of the sandwich beam in the contact area |
| $\dot{\Delta}$ | global sandwich beam velocity |
| $\Delta_f$ | global displacement of the sandwich beam due to bending moment |
| $\Delta_s$ | global displacement of the sandwich beam due to shear forces |
| $\nu_{12}$ | principal Poisson's ratio |
| $\xi$ | distance outside the contact area |
| $\Pi$ | potential energy |
| $\rho_c$ | density of the core material |
| $\rho_f$ | density of the face-sheet |
| $\rho_s$ | density of the sandwich beam |

The aim of this work is to develop a non-dimensional analytical model to predict the dynamic response of composite sandwich beams subjected to low-velocity impact, prior to visible failure of the upper face-sheet. Experimental three-point bending tests were conducted to validate the model predictions. A dimensional analysis was carried out to group the key parameters of the model as dimensionless groups, and study their influence on the contact force and contact time.

## 2. Model formulation

The model developed in this work is based on Hoo Fatt and Park model [15]. The proposed model allows the analysis of the impact response of composite sandwich beams subjected to low-velocity impact, considering the effect of the nonlinear relationship between the indentation force and the local displacement of the upper face-sheet in the formulation. The effects of the inertial masses of both the upper face-sheet and the whole sandwich structure are included. Low-velocity impact tests on simply-supported sandwich beams using a cylindrical impactor (Fig. 1a) were conducted to validate the predicted results. The model is formulated in terms of dimensionless parameters in order to determine the key dimensionless groups which control the dynamic response of the sandwich beams. The formulation of the model leads to a system of nonlinear differential equations which cannot be solved analytically, and therefore requires the use of numerical methods.

The problem is modelled as a discrete system of two-degrees-of-freedom (Fig. 1b). In Fig. 1b, the global stiffness of the beam is represented as a linear spring $K_g$ whereas to represent the local contact between the upper face-sheet and the impactor, a nonlinear local spring is employed $K_l$. $M_0$ is the mass of the impactor which contacts the upper face-sheet of the sandwich structure. The inertia of the upper face-sheet is represented by an effective mass $m_f$, and the inertia of the mass of the sandwich beam is represented by and effective mass, $m_s$. The core crushing load is represented by $Q_d$. The local displacement of the upper face-sheet and the global displacement of the whole sandwich structure are represented by $\delta$ and $\Delta$, respectively.

The equations of motion for the two-degrees-of-freedom system in Fig. 1b are defined as:

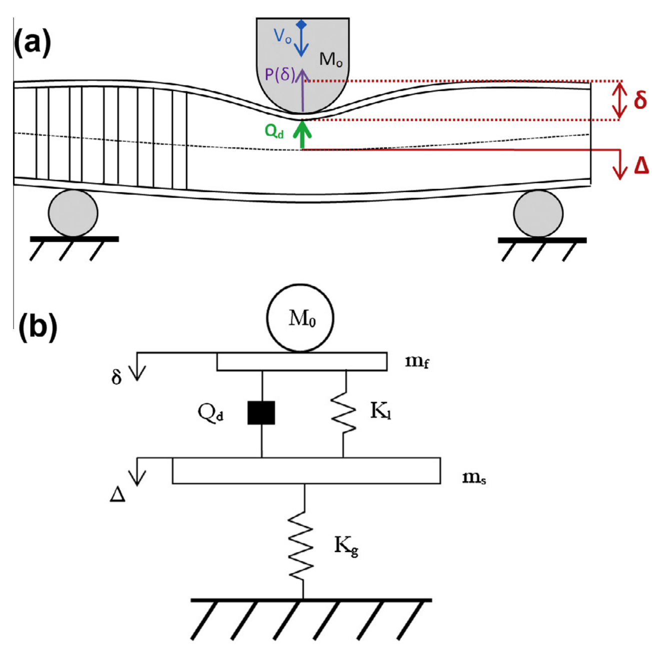


**Fig. 1.** Low-velocity impact on a simply-supported sandwich beam: (a) schematic representation of the problem, (b) two-degree-of-freedom mass-spring model.