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Buckling and free vibration of magnetoelectroelastic nanoplate based on nonlocal theory

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ABSTRACT

Buckling and free vibration of magnetoelectroelastic nanoplate resting on Pasternak foundation is investigated based on nonlocal Mindlin theory. The in-plane electric and magnetic fields can be ignored for nanoplates. According to Maxwell equations and magnetoelectric boundary conditions, the variation of electric and magnetic potentials along the thickness direction of the nanoplate is determined. Using the Hamilton's principle, the governing equations of the magnetoelectroelastic nanoplate are derived. Numerical results reveal the effects of the electric and magnetic potentials, spring and shear coefficients of the Pasternak foundation on the buckling load and vibration frequency. These results can serve as benchmark solutions for future numerical analyses of magnetoelectroelastic nanoplates.

1. Introduction

Nanostructures have increased considerable attention among the experimental and theoretical research communities. These nanostructures [1–3] are found to be possessing extraordinary mechanical, electrical, electronics and thermal properties as compared to the conventional structural materials. A vast area of novel applications of these nanostructures is foreseen in the coming years. These include aerospace, biomedical, bioelectrical, superfast microelectronics, etc. Understanding the accurate mechanical and physical properties of these nanostructures and their impacts on its performance and reliability is thus necessary for its productive applications. Therefore, microstructure-dependent size effects are often observed [4–8].

In the domain of materials science, some recent advances are the smart or intelligent materials where piezoelectric and piezomagnetic materials are involved. These materials called magnetoelectroelastic composites have the ability of converting energy from one form (among magnetic, electric, and mechanical energies) to the other. Furthermore, they exhibit magnetoelectric effect that is not present in single-phase piezoelectric or piezomagnetic materials [9–14].

Recently, much attention has been paid to the structural analysis of the magnetoelectroelastic plate. Pan [15] presented an exact closed-form solution for the static deformation of the layered piezoelectric/piezomagnetic plate based on a new and simple formalism resembling the Stroh formalism, and the propagator matrix method was used to handle the multilayered case. Using the state vector method, Wang et al. [16] obtained an analytical solution for magneto-electro-elastic, simply supported and multilayered rectangular plates in the form of infinite series. The state-vector approach was proposed by Chen et al. [17] for the analysis of free vibration of magneto-electroelastic layered plates. Wang et al. [18] derive the analytical solution for a three-dimensional transversely isotropic axisymmetric multilayered magneto-electro-elastic (MEE) circular plate under simply supported boundary conditions. Liu and Chang [19] presented the closed form for the vibration problem of a transversely isotropic magneto-electro-elastic plate. A nonlinear large-deflection model for magnetoelectroelastic rectangular thin plates is proposed by Xue et al. [20]. The bending problem for a transversely isotropic MEE rectangular plate is analyzed by imposing the Kirchhoff thin plate hypothesis on the plate constituent. An equivalent single-layer model for the dynamic analysis of magnetoelectroelastic laminated plates is presented by Milazzo [21]. The electric and magnetic fields are assumed to be quasi-static and the first-order shear deformation theory is used.

Considered the nonhomogeneous magnetoelectroelastic solids, Bhangale and Ganesan [22] carried out static analysis of FGM magneto-electro-elastic plate by finite element method under mechanical and electrical loading. Wu et al. [23] extended the Pagano method for the three-dimensional plate problem to the analysis of a simply-supported, functionally graded rectangular plate under magneto-electro-mechanical loads.

To the best of authors' knowledge, however, the buckling and free vibration of magnetoelectroelastic nanoplate resting on a Pasternak foundation has not been considered.

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Based on the nonlocal theory, the buckling and free vibration analysis of a magnetoelectroelastic nanoplate resting on a Pasternak foundation is investigated. The in-plane electric and magnetic fields can be ignored for nanoplates. According to Maxwell equations and magnetoelectric boundary conditions, the variation of electric and magnetic potentials along the thickness direction of the nanoplate is determined. The governing equations of magnetoelectroelastic nanoplate are derived based on application of Hamilton's principle. Numerical results reveal the effects of the electric and magnetic potentials, spring and shear coefficients of the Pasternak foundation on the buckling load and natural frequency.

2. Nonlocal and Mindlin plate theories

2.1. Nonlocal theory of magnetoelectroelasticity

Nonlocal elastic theory assumes that the stress state at a reference point x in the body is regarded to be dependent not only on the strain state at x but also on the strain states at all other points x' of the body. The most general form of the constitutive relation in the nonlocal elasticity type representation involves an integral over the entire region of interest. The integral contains a nonlocal kernel function, which describes the relative influences of the strains at various locations on the stress at a given location. The constitutive equations of linear, homogeneous, isotropic, non-local elastic solid are given by Eringen [24].

$$\sigma_{ij}^{nl}(x) = \int_v \alpha(|x-x'|,\tau)\sigma_{ij}^{l} dV(x'), \quad \forall x \in V, \tag{1}$$

where σ_{ij}^{nl} and σ_{ij}^{l} are, respectively, the nonlocal stress tensor and local stress tensor. $\alpha(|x-x'|,\tau)$ is the nonlocal modulus, $|x-x'|$ is the Euclidean distance, and $\tau = e_0a/l$ is defined that l is the external characteristic length, e_0 denotes a constant appropriate to each material, and a is an internal characteristic length of the material. Consequently, e_0a is a constant parameter which is obtained with molecular dynamics, experimental results, experimental studies and molecular structure mechanics. For the magnetoelectroelastic solid, the nonlocal constitutive equation can extend to Eq. (1) and the following equations

$$D_k^{nl}(x) = \int_v \alpha(|x-x'|,\tau)D_k^{l} dV(x'), \quad \forall x \in V, \tag{2}$$

$$B_k^{nl}(x) = \int_v \alpha(|x-x'|,\tau)B_k^{l} dV(x'), \quad \forall x \in V, \tag{3}$$

where D_k^{nl} and D_k^{l} are the components of the nonlocal and local electric displacement, respectively, and B_k^{nl} and B_k^{l} are the components of the nonlocal and local magnetic induction, respectively.

Making certain assumptions presented by Eringen [24,25], the integral equations of nonlocal elasticity can be simplified to partial differential equations. Eqs. (1)–(3) takes the following simple form:

$$\left(1-(e_0a)^2\nabla^2\right)\sigma_{ij}^{nl} = \sigma_{ij}^{l}, \tag{4}$$

$$\left(1-(e_0a)^2\nabla^2\right)D_k^{nl} = D_k^{l}, \tag{5}$$

$$\left(1-(e_0a)^2\nabla^2\right)B_k^{nl} = B_k^{l}. \tag{6}$$

where ∇^2 is the Laplacian operator in the above equations.

2.2. The Mindlin plate theory

Based on the Mindlin plate theory, the displacement field can be expressed as

$$u_x(x,y,z,t) = z\psi_x(x,y,t), \tag{7}$$

$$u_y(x,y,z,t) = z\psi_y(x,y,t), \tag{8}$$

$$u_z(x,y,z,t) = w(x,y,t), \tag{9}$$

where $\psi_x(x,y,t)$ and $\psi_y(x,y,t)$ are the rotations of the normal to the mid-plane about x and y directions, respectively.

The non-zero strain associated with the above displacement field can be expressed in the following form:

$$\varepsilon_{xx} = z\frac{\partial\psi_x}{\partial x}, \tag{10}$$

$$\varepsilon_{yy} = z\frac{\partial\psi_y}{\partial y}, \tag{11}$$

$$\gamma_{yz} = \frac{\partial w}{\partial y} + \psi_y, \tag{12}$$

$$\gamma_{zx} = \frac{\partial w}{\partial x} + \psi_x, \tag{13}$$

$$\gamma_{xy} = z\left(\frac{\partial\psi_x}{\partial y} + \frac{\partial\psi_y}{\partial x}\right), \tag{14}$$

where ε_{xx}, ε_{yy} are the normal strain components and γ_{yz}, γ_{zx}, γ_{xy} are the shear strain components.

3. Modeling of the problem

Consider a magnetoelectroelastic nanoplate with length l, width b and thickness h resting on a Pasternak foundation as depicted in Fig. 1. A Cartesian coordinate system (x,y,z) is used to describe the plate with z along the plate thickness direction and the $x-y$ plane sitting on the midplane of the undeformed plate. The magnetoelectroelastic body is poled along z-direction and subjected to an electric potential V_0 and a magnetic potential Ω_0 between the upper and lower surfaces of the plate.

3.1. Constitutive relations for magnetoelectroelastic nanoplate

$$\begin{Bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{yz}\\ \sigma_{zx}\\ \sigma_{xy}\end{Bmatrix} = \begin{bmatrix} c_{11} & c_{12} & 0 & 0 & 0\\ c_{12} & c_{22} & 0 & 0 & 0\\ 0 & 0 & c_{44} & 0 & 0\\ 0 & 0 & 0 & c_{44} & 0\\ 0 & 0 & 0 & 0 & c_{66}\end{bmatrix}\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{yz}\\ \gamma_{zx}\\ \gamma_{xy}\end{Bmatrix} - \begin{bmatrix} 0 & 0 & e_{31}\\ 0 & 0 & e_{31}\\ 0 & e_{24} & 0\\ e_{15} & 0 & 0\\ 0 & 0 & 0\end{bmatrix}\begin{Bmatrix}E_x\\ E_y\\ E_z\end{Bmatrix} - \begin{bmatrix} 0 & 0 & f_{31}\\ 0 & 0 & f_{31}\\ 0 & f_{24} & 0\\ f_{15} & 0 & 0\\ 0 & 0 & 0\end{bmatrix}\begin{Bmatrix}H_x\\ H_y\\ H_z\end{Bmatrix}, \tag{15}$$

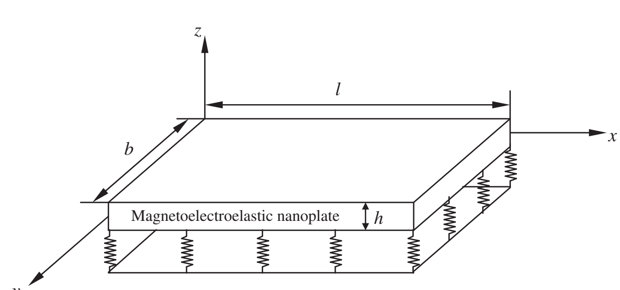

Fig. 1. Schematic of a magnetoelectroelastic naoplate resting on Pasternak elastic foundation.