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Buckling analysis of restrained orthotropic plates under combined in-plane shear and axial loads and its application to web local buckling

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ABSTRACT

Buckling analysis of orthotropic plates with two opposite edges simply supported and the other two opposite edges rotationally-restrained (RR) and under combined uniform in-plane shear and linearly varying axial loads is presented, and its application to web local buckling of composite structural shapes is illustrated. A new plate buckled displacement shape function is proposed, and the approximate solution is obtained by the Rayleigh–Ritz method. The accuracy of analytical solution is validated with the numerical finite element analysis, and excellent agreements are achieved. A parametric study is conducted to evaluate the influence of loading ratio and rotational restraint stiffness. By introducing generic non-dimensional parameters, the buckling formulas of long plates under uniform in-plane compression, pure in-plane bending and uniform in-plane shear are obtained using the curve fitting technique. Interaction curves between the uniform in-plane shear and pure in-plane bending for the simply supported (SS) and clamped-simply supported (CS) plates are established, and it is found that the interaction curve is only related to the material orthotropic parameter of β. Finally, the proposed discrete restrained plate solution is applied to predict the web local buckling of FRP shapes by adopting the proper rotational restraint stiffness.

1. Introduction

Common fiber-reinforced plastic (FRP) composite structural shapes are thin-walled sections consisting of flat panels [1–3]. Due to the high specific strength, high specific stiffness, high flexibility in design, etc., the application of FRP shapes has expanded from the traditional areas (e.g., aerospace engineering [4–6]) to new ones (e.g., shipbuilding, civil infrastructure [3,7,8], and so on). However, problems associated with large deformation and local buckling are common before the strength failure in material is reached, due to their relatively low stiffness to strength ratio and thin-walled configuration. Therefore, buckling is often the governing failure mode in analysis and design of FRP structural shapes [7]. One of the simple methods for local buckling analysis of thin-walled FRP structural shapes is the discrete plate analysis technique [1,3,9], in which the plate elements in the thin-walled member are individually analyzed with consideration of boundary restraints from the adjacent plate element(s).

The study on local buckling of plates with various edge restraints and under axial loads has already attracted great attentions. The buckling solutions and parametric studies of an orthotropic plate under uniform compression, simply supported on its loaded edges and free and rotationally-restrained on its unloaded edges were discussed by Bank and Yin [10]. And the closed form solutions were derived for the buckling loads of this kind of orthotropic plates by Kollar [11]. The closed form approximate formulas were presented for the calculation of the buckling load of rectangular orthotropic plates with clamped (C) and/or simply supported (S) edges by Veres and Kollar [12]. The explicit analytical solutions for orthotropic plates with various unloaded edge boundary conditions and uniform compression were obtained by Qiao and his co-workers [2,3,13–15]. Considering the flexural-twist anisotropy, the closed-form solution for long plates under axial compression was presented by Weaver and his co-workers [16,17]. The approximate formulas for the long rectangular orthotropic plates with simply supported or clamped edges and subjected to uniform axial compression, uniform shear, or pure in-plane bending loads were provided by Weaver and Nemeth [18]. An exact buckling solution of plates under linearly varying axial loads was obtained as a power series (i.e., the method of Frobenius) by Kang and Leissa [19], and it

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was used to the buckling analysis of rectangular plates with two opposite edges simply supported, and the other two edges clamped, simply supported, free, or elastically supported. An exact solution was also presented for buckling of simply supported symmetrical cross-ply rectangular plates under linearly varying loads [20]. The closed-form analysis of the buckling behavior of orthotropic plates with elastic clamping and edge reinforcement under uniform compressive load was presented by Weissgraeber et al. [21], and the restraint coefficient is different from those given in [9,22,23].

The buckling study on plates under uniform shear and linearly varying axial loads is also an important topic, and its solution can be applied to simplified local buckling analysis of shear-dominant webs in the thin-walled structures. The exact buckling solution of orthotropic plates with the short edges simply supported and long edges restrained by the rotational springs under uniform shear using static method was developed by Seydel [24], and the approximate formulas for the critical loads and aspect ratios of plates with the two longitudinal edges simply supported or clamped were derived. The buckling problems of isotropic and orthotropic plates under uniform shear were studied by Johns [25]. The buckling solutions for isotropic Mindlin plates under uniform shear were obtained using the Rayleigh–Ritz method [26]. Comprehensive analysis to the buckling problems of general long laminates subjected to various loads was presented by Nemeth [4–6]; however, there are only solutions for plates with the longitudinal edges either clamped or simply supported. By simulating the buckled displacement shape function as a trigonometric series [1], the problem of buckling of plates with two opposite edges simply supported and the other two edges elastically-supported and under uniform in-plane shear was numerically solved. Using the same buckled shape function, the numerical solutions for the buckling problem of long plates under linearly varying axial load and in-plane shear were obtained, and the simple expressions of local buckling loads were obtained by curve-fitting the numerical solutions [22]. The simple expressions developed were then used to predict the web buckling of composite structural shapes under various transverse loads [23]. An explicit closed-form local buckling solution was presented for the plates under uniform shear with two opposite edges simply supported and the other two opposite edges either both rotationally-restrained (the so-called RR plates) or one rotationally restrained and the other free (the so-called RF plates) [27].

When in the process of calculating the buckling loads, it is found that the plate buckled shape functions adopted in most of the existing studies [1,7,22] only satisfy the essential boundary conditions but not the natural (force) boundary conditions which are not necessary. Therefore, the convergence rate of the solution is slow, and it has to adopt more terms to obtain a reasonably accurate numerical solution. In addition, when the rotational restraint stiffness approaches infinite (i.e., to the clamped case), more terms of approximation in the shape functions are required. Furthermore, it has to exclude the case of clamped boundary conditions in which the rotational restraint stiffness tends to infinite. While the buckling load previously obtained by Qiao and Zou [2] for long plates subjected to a special case of linearly varying load (i.e., the pure in-plane bending load) along the short edges resulted in an infinite one, which is unrealistic. Furthermore, for thin-walled structures under transverse loads, the rotational restraint stiffness of the top and bottom edges of the web is not necessarily the same. Thus, there is a need to develop relatively simple yet realistically accurate local buckling solution for the restrained orthotropic plates under combined in-plane shear and linearly varying axial loads and apply such a solution for local web buckling analysis of FRP structural shapes.

Recently, buckling behavior of hybrid pultruded FRP short columns under uniform pressure were studied by Correia et al. [28], and a particular attention was given to the local buckling. Global and local buckling behavior of pultruded glass FRP columns subjected to small eccentric loading were characterized using the combined experimental and numerical methods by Nunes et al. [29]. The influence of boundary conditions and geometric imperfections on the lateral-torsional buckling resistance was studied using the nonlinear method by Nguyen et al. [30]. Using the discrete plate analysis, analysis and design guidelines of FRP structural shapes were presented in the literature [1–3,7,9,13,22,31]. It is worth mentioning that in the web local buckling analysis [7,23] the uniform shear through the width and unchanged normal axial stress along the longitudinal direction of the representative discrete web plate element were assumed though the shear stress distribution in the web are often parabolic and the normal stress varies along the longitudinal direction of the beam.

In this paper, based on the Rayleigh–Ritz method, the local buckling analysis of a thin orthotropic plate with two opposite edges simply supported and the other two opposite edges rotationally-restrained and under uniform in-plane shear and linearly varying axial loads is presented. A relatively simple buckled displacement shape function is proposed, and it does satisfy not only the essential boundary conditions but also the natural boundary conditions. By using the constructed shape function, the buckling analysis of plate under the combined uniform in-plane shear and linearly varying axial loads becomes easier. The solutions are verified by comparing with available solutions in the literature and numerical finite element analysis. A parametric study is also conducted to evaluate the effect of several parameters, such as the load ratios, rotational restraint stiffness, and flexural-orthotropy, on the buckling coefficients of infinitely long plates, and the approximate explicit closed-form formulas for a few simplified boundary cases are obtained as well. Moreover, a detailed study of interaction curve between uniform in-plane shear and pure in-plane bending loads is conducted. Finally, the analytical plate solution is applied to analyze the web local buckling of FRP shapes under various loads using the discrete plate analysis techniques and considering some simplified assumptions of loads acting on the web plates.

2. Generalized buckling analysis

2.1. Rayleigh–Ritz method

Structural members in reality may be subjected to a variety of shear, bending, and axial loading. For a relatively long plate (e.g., the plate aspect ratio $\gamma = a/b \geqslant 5$), the buckling load asymptotically approaches to a constant value [1,7]. For example, the axial load produces a repeating buckling pattern with an asymptotic buckling strength as the length of the plate increases; while similarly, the in-plane shear buckling strength decreases as the length increases, but the rate of change is very small and an asymptotic value can still be assumed for $\gamma \geqslant 5$. Therefore, it is possible to extract a representative plate element (as shown in Fig. 1) employing the periodic geometric properties with a half buckled wave length.

Among all the methods of the appropriate determination of critical loads, the Rayleigh–Ritz method is an important and effective one, and it is based on the general theorem of the equilibrium of mechanical systems. In the representative plate element shown in Fig. 1, the total potential energy (Π) consists of three parts: (1) the elastic strain energy (U_e) stored in the orthotropic plate; (2) the restraining energy (U_r) stored in the rotational restraining springs or torsional stiffeners at the plate edges; and (3) the work (W) done by the external forces, and it is expressed as