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Derivation and experimental validation of Lamb wave equations for an *n*-layered anisotropic composite laminate

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ABSTRACT

Lamb waves are ultrasonic guided waves that propagate between two parallel free surfaces and their use for damage detection has been widely explored and demonstrated. Damage in materials/structures can be detected by analyzing the difference between the phase/group velocity and the loss of amplitude of Lamb waves on damaged and un-damaged specimens. The propagation characteristics of Lamb waves are described in the form of dispersion curves, which are plots of phase/group velocities versus the product of frequency-thickness generated by solving the Lamb wave equations. Lamb waves' dispersion behaviors for isotropic materials are well established in the literature; however, such is not the case for the laminated composites. The most common methods for solving the Lamb wave equations in composites consist of using laminated plate theory or 3D linear elasticity by assuming an orthotropic and/or higher symmetry. This assumption may not be true, if the actuators and sensors in an orthotropic or transversely isotropic laminates are installed in a non-principle direction or the layup is symmetric but not balanced.

This paper presents a full derivation of Lamb wave equations for *n*-layered monoclinic composite laminates based on linear 3D elasticity by considering the displacement fields in all three directions using the partial wave technique in combination with the Global Matrix (GM) approach. In the partial wave technique, the principle of superposition of three upward and three downward travelling plane waves are assumed in order to satisfy the associated boundary conditions. The bounded upper and lower surfaces reflect the waves and the combination of these reflections going towards the upper or lower interfaces results in the propagating guided waves. The GM approach is used to assemble all the equations from each layer to form a global, unified matrix that describes the displacement and stress fields along the entire laminate associated with the wave propagation. A robust method for numerically solving the Lamb wave equations is also presented.

The presented method was verified experimentally by analyzing the propagation of Lamb waves in two different composite panels constructed out of unidirectional carbon-fiber epoxy prepreg and fiber-metal laminate (GLARE 3-3/4). The panels were instrumented with *lead zirconate titanate* (PZT) piezoelectric sensors, which were excited at different frequencies ranging from 20 kHz to 500 kHz to generate and acquire the waves. The waves were excited and gathered at three different propagation angles of 0°, 45°, and 90° for the carbon-fiber epoxy laminate panel and at six different angles of 0°, 20°, 45°, 70° and 90° for the fiber-metal laminates (GLARE). The phase and group velocities of the fundamental symmetric (S_o) and anti-symmetric (A_o) Lamb waves were extracted by tracking the peaks of each individual wave phase and the wave envelope respectively using an in-house code developed in MATLAB. It was found that the presented 3D linear elasticity model followed the experimental data closely for both symmetric and anti-symmetric Lamb modes. The analytical method presented in this paper was able to predict the Lamb wave dispersion for both the carbon-fiber epoxy laminate and the hybrid fiber-metal laminate proving the robustness and versatility of the solution method.

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1. Introduction

The existence of Lamb waves was originally proven mathematically by Lamb [1], based upon the Rayleigh wave equations developed by Rayleigh [2]. The difference between Rayleigh and Lamb waves are the boundary conditions affecting their propagation characteristic. Rayleigh waves propagate in a medium with one nearby free surface boundary; whereas the propagation of Lamb waves is guided by two nearby parallel free surface boundaries. Lamb waves are elastic perturbations resulting from the superposition of longitudinal waves (P-waves) and shear waves (S-waves), for which the displacements occur both in the direction of wave propagation and perpendicular to it [3]. Compared with body waves, which propagate in solids, far away from the free surface boundaries, Lamb waves can propagate through longer distances with minimal loss of energy and therefore low amplitude attenuation. This enables the detection of Lamb waves with reasonable *signal-to-noise ratios* (SNR) even in highly dispersive/attenuative materials such as polymer matrix composites. The attenuation is attributed to the small damping imposed by the two close-by parallel free surface boundaries, which can be observed in a plate or shell-like structural elements. One interesting aspect of Lamb waves is that their propagation within the host material is influenced by the presence of flaws and damage, as it is by any other boundary condition. This enabled the application of Lamb waves for damage detection in *Non-Destructive Inspection* (NDI) techniques, which was first proposed by Worlton [4]. After the initial experiments performed by Frederik and Worlton [5], the application of Lamb waves has been widely explored, particularly when related to smart structures with permanently embedded, or bonded sensors, as envisioned by the *Structural Health Monitoring* (SHM) community. The high number of proposed, experimented, and demonstrated applications is related with the advance in electronics and computational capability enabling the generation, acquisition, and processing of Lamb waves even at high frequencies in the MHz range.

For the intended application of Lamb waves, the understanding of the underlying physics behind the Lamb waves and their multi-mode propagation characteristics within the host material is essential. Lamb waves can exist simultaneously in two modes, which are symmetric (S_n) and anti-symmetric (A_n) modes, that can propagate independently of each other. Here, the subscript (n) is an integer indicating the order of the mode, or the number of inflexion points found in the wave deformation field across the thickness. For a finite plate thickness there exist an infinite number of such multi-mode symmetric and anti-symmetric Lamb waves, along with shear horizontal (SH_n) waves, differing from one another by their phase and group velocities as well as distribution of the displacements and stresses through the plate thickness [6]. However, for most practical applications only input signals that excite uniquely the fundamental (with no inflection points) anti-symmetric (A_o) and symmetric (S_o) Lamb waves are usually considered to avoid an increased complexity in the interpretation of gathered wave signals containing interference of the multiple modes. Besides the existence of several wave modes, Lamb waves are dispersive, i.e., the propagation velocity of each wave mode and their order/excitation depends on the excitation frequency. This behavior is predicted by the relevant Lamb wave characteristic equations and is represented by the dispersion curves, i.e., propagation velocity versus frequency-thickness curves. The dispersive behavior and the characterization of the displacement fields of the different waves and modes are essential for the selection of sensors, sensor dimensions/types, installation positions on/in the structure, associated sensor systems, signal generation and acquisition, and data processing [7].

Propagation characteristics of Lamb waves for isotropic materials are well defined in the literature from Mindlin [8], Viktorov [3] to Rose [9], which is not the case for composites. The use of composite is growing rapidly in the aerospace structures; where there is a strong desire to use materials with high strength to weight ratios. Furthermore, the use of composite enables the application of tailored designs by applying the appropriate strength/stiffness in the required directions, while minimizing the structural weight. However, composite materials present radically different behaviors as response to damage existence as compared to their better understood metallic (isotropic) counterparts. In some cases this behavior is not desirable, since damage can grow rapidly and/or cannot be easily detected using the conventional NDI methods. Lamb waves application has shown some promise for damage detection in composite structures [10–12], however; as mentioned previously, most of the theoretical analysis development enabling the precise application of Lamb waves has been performed mostly for isotropic materials, or for anisotropic materials with orthotropic and higher symmetry.

Propagation of Lamb waves in composites are complex due to material anisotropy and strongly attenuative/dispersive behavior of the wave [13]. Parameters of composites materials such as fiber volume fractions, layup sequence, and types of matrix/reinforcements used, strongly influence the wave propagation characteristics. Waves in composite plates propagate in each direction with different velocities, with the shape of the wave front changing with frequency [6]. For simplification, composite laminates are assumed to have orthotropic or higher degrees of symmetry to generate the dispersion curves. The simplest method to generate the Lamb wave dispersion curves as compared with other methods is by using the effective stiffness approach, in which the geometrically weighted average of the constituent properties are used as the average material constants for the entire laminate [14]. Another simpler method is by using the *classical laminated plate theory* (CLPT); however, the CLPT fails to accurately predict the Lamb wave dispersion characteristic at higher frequencies [15]. Therefore, the CLPT has been preceded by higher-order plate theories [16] to better predict the dispersion characteristics. Despite being computationally efficient, CLPT and higher-order theories are only an approximation and fail to accurately predict the higher Lamb wave modes at higher frequencies [17]. Datta et al. [18] provided another approximation method for a multi-layered transversely isotropic material based on stiffness method [19] in which the displacement distribution through the thickness was approximated by polynomial interpolation functions. An additional hybrid method includes the use of *Semi-Analytical Finite Element* (SAFE), which uses the *Finite Element Method* (FEM) to discretize the cross-section and describes the displacement along the wave propagation with the use of analytical simple harmonic functions [20]. To date the exact method for characterizing Lamb waves' propagation in composites is by using the linear 3D elasticity method. Some of the noticeable work regarding the dispersion relationship is provided in the following paragraph.

Anderson [21] used a curl-free and divergence-free displacement vector field approach in a single layer with hexagonal symmetry to derive the dispersion curves and presented a method based on Hankel [22] to extend his approach for a multi-layer composite. Solie and Auld [23] used the partial wave technique for cubic symmetry using Mindlin boundary conditions [24] to obtain the decoupled shear-vertical (SV) and longitudinal (P) modes dispersion curves. Kennet [25] provided a derivation for the coupling between the P and SV waves for a stratified isotropic medium. Chimenti and Nayfeh [26] presented leaky Lamb wave equations for a unidirectional composite laminate with transversely isotropic