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Analysis of particle–matrix interfacial debonding using the proper generalized decomposition

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ABSTRACT

Syntactic foams are a class of particulate composites consisting of microballoons dispersed in a matrix material. While several modeling schemes have been developed to study their elastic response, the mechanics of failure of these composites is a largely untapped research area. Here, we propose a mathematically tractable framework to analyze particle–matrix interfacial debonding in uniaxial tension. The proper generalized decomposition is used to study the deformation of the matrix and the inclusion, and the method of Lagrange multipliers is adapted to satisfy the boundary conditions along the bonded portion of the inclusion–matrix interface. A variational approach is utilized to derive the governing differential equations, and the Galerkin method is implemented to cast the problem into a manageable set of algebraic equations. An iterative procedure based on the fixed point algorithm is ultimately used to determine the displacement fields. Results are specialized to a glass particle–vinyl ester matrix system, and a parametric study is conducted to understand the mechanics of debonding. Results are validated through available data and new finite element simulations. We find that the proposed framework is in very good agreement with numerical results for a wide range of debonding angles, inclusion volume fractions, and inclusion wall thicknesses.

1. Introduction

Syntactic foams are lightweight composite materials that are obtained by embedding thin hollow inclusions, often called microballoons, in a matrix material [39]. The hollow reinforcements allow for improving stiffness and strength [7,21,22,37,39,45,52,68], reducing structural weight [23,29,62], and controlling moisture absorption and thermal expansion [48,57,65].

Due to the wide spectrum of applications of syntactic foams as core materials for sandwich panels in marine and aerospace structures [24,30,46], several approaches have been proposed in the literature to predict their elastic properties [7,8,26,36,42,68]. These studies have helped identifying the role of the inclusion wall thickness and volume fraction on the Young's modulus and Poisson's ratio of syntactic foams. In particular, Huang and Gibson [26] utilized an infinitely dilute inclusion dispersion to predict syntactic foams' elastic properties. Bardella and Genna [8] proposed a four-phase unit cell model based on a self-consistent scheme to study syntactic foams' elastic response, and a similar multi-phase concentric sphere model was utilized by Marur [36]. Porfiri and Gupta [42] developed a differential scheme to analyze the elastic properties of syntactic foams at high inclusion volume fraction. In [10], a thorough review of these approaches was presented along with a detailed assessment against finite element simulations on numerical models comprising fifty inclusions.

Only recently, some research effort have been devoted to the analysis of the failure mechanisms of syntactic foams [9,31,37,49–51,55,56]. More specifically, Jones et al. [31] and Shams et al. [49] studied the static buckling of a microballoon embedded in an infinite elastic medium under remote uniaxial loading to shed light on the phenomenon of inclusion crushing in syntactic foams' compression. A comparison of different shell theories and computational methods for the prediction of the compressive failure of isolated microballoons has been recently presented in [51]. In [9], the compressive failure of a micromechanical model with fifty inclusions was studied through finite element simulations. In [55,56], particle–matrix debonding was analyzed in a unit cell model, consisting of a partially debonded hollow inclusion in an infinitely extended matrix. Debonding was modeled through two interfacial spherical cap cracks symmetrically located along the tensile loading direction, and the problem is analyzed using three dimensional elasticity. An alternative approach aimed at reducing the computational complexity has been recently introduced by [50]. Therein, the generalized Vla-

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sov–Jones foundation model and Donnell linear shell theory were used to model the matrix and the inclusion, respectively. The comparison with finite element results reported in [50] shows that the method is successful in predicting the energy storage in the unit cell only for relatively small interfacial cracks, while significant discrepancies are observed as the extent of the debonds exceeds a few percent of the particle surface. As described therein, this is likely due to the simplicity of the foundation model, which does not allow to accurately describe the complex dependence of the matrix deformation on both the radial and meridional coordinates, as observed in [55,56]. The main objective of this paper is to establish an effective modeling framework to alleviate the drawbacks reported in [50] in the description of the matrix deformations through the use of the proper generalized decomposition (PGD) [4,15,61].

The PGD is widely employed to study multi-dimensional [4,5,13,15,17,20,41], multi-physics [2,33,40], and degenerate geometrical domain problems that are derived from three-dimensional geometries such as plates and shells [59–61]. The PGD is based on the "separated representation", which consists of using successive enrichments to construct an approximation of the solution in the form of a finite sum of the product of functions of the system coordinates [15,60,61]. The separated representation was originally proposed by Ladevéze [32] to solve a nonlinear transient thermo-mechanical problem within the large time increment (LATIN) method. The PGD scheme was adapted to solve multi-dimensional transient Fokker–Planck equation in space–time domain in [4,5]. For degenerate geometrical applications, in [59–61], the PGD was used to model laminated and sandwich structures based on the finite element method, similarly to the approach presented in [13]. Results reported in [59–61] were compared to a classical layer-wise solution of the problem in terms of different geometries, loading conditions, and boundary conditions to assess the potential of the approach. Giner et al. [20] studied the problem of a three-dimensional crack in a linear elastic plate by considering Poisson's ratio and plate thickness as additional unknowns in a separated representation solution of the problem using the PGD. Analyses reported in [20] demonstrate that the PGD drastically reduces the computational complexity and decreases the computational costs, even if material and geometry parameters are considered as independent variables in the solution [13,15,17]. In addition, Ladevéze et al. [33] simultaneously utilized the LATIN method and the PGD to study a three-dimensional L-shape substructure under a distributed load with viscoelastic material properties. In [15,16,41], extensive reviews on the applications, foundations, and model reduction properties of the PGD can be found..

In this work, we use the PGD to describe the displacement fields in the unit cell problem considered in [50]. The unit cell, constituted by a spherical matrix embedding a linear Donnel thin shell [3,53] is subjected to uniaxial tension load at the outer radius of the matrix. We consider two equal spherical cap cracks along the inclusion–matrix interface, at the north and south poles of the inclusion. Lagrange multipliers are used to impose the essential boundary conditions along the bonded portion of the inclusion and the matrix [12,67]. We derive the governing equations of the unit cell using a variational approach, and the Galerkin method is adapted to derive a set of coupled nonlinear algebraic equations to describe the until cell deformations. An iterative method based on the fixed point algorithm [61] is implemented to find the displacement fields in the unit cell. We conduct a parametric study to elucidate the effect of shell wall thickness, debonding angle, and volume fraction on the displacement fields, energy storage, and opening displacement at the inclusion–matrix interface. The proposed framework is validated through available data in the literature [50,55] and new finite element results.

The paper is organized as follows. In Section 2, we define the problem geometry, and the kinematic assumptions of the shell and the matrix are introduced. In Section 3, the displacement fields in the shell and the matrix are described based on the PGD. In Section 4, we derive the governing equations of the problem using a variational method. Therein, we present an iterative method to solve the sets of the coupled nonlinear algebraic equations. In Section 5, the analyses are specialized to a glass particle–vinyl ester matrix system, and a parametric study is performed to verify the accuracy of the proposed scheme. In Section 6, the main conclusions of this study are summarized. Further, in Appendix A, we outline the residual equations used in Section 4.2. In Appendix B, we report the coefficients used in the governing equations and the matrix components used in Section 4.2. In Appendix C, details on the finite elements are described. Finally, in Appendix D, we report on the convergence study of the analytical solution.

2. Problem statement

We analyze a unit cell consisting of a single hollow inclusion with mean radius R_s and wall thickness h embedded in a spherical matrix with outer radius R_m. The volume fraction of the inclusion is defined as $\Phi = (R_s/R_m)^3$. We consider a spherical coordinate system (r, θ, ϕ) to describe the problem, where r is the radial coordinate, θ is the meridional direction (zenith angle), and ϕ is the circumferential direction (azimuth angle). Fig. 1 shows the spherical coordinates along with the Cartesian coordinate system (x, y, z). In this problem, the north and south poles of the inclusion are the points at $y = \pm R$, respectively, that is, $\theta = 0$ and $\theta = \pi$ (see Fig. 1). Similarly, the equator of the inclusion is at $y = 0$, that is, $\theta = \pi/2$. The inclusion and the matrix are described by subscripts s and m, respectively. We assume the presence of two equal spherical cap cracks at the north and south poles of the inclusion–matrix interface following [50,55]. The cracks' extension angles on the inclusion–matrix interface are described by θ_0. The constituents' materials are assumed to be linear, elastic, isotropic, and homogenous. The system is subjected to uniaxial tension loading in the y-direction. The symmetry of the loading conditions with respect to the y-axis imposes that the displacement in the circumferential direction is zero and that the radial and the meridional components of the displacement are independent of ϕ. We further utilize

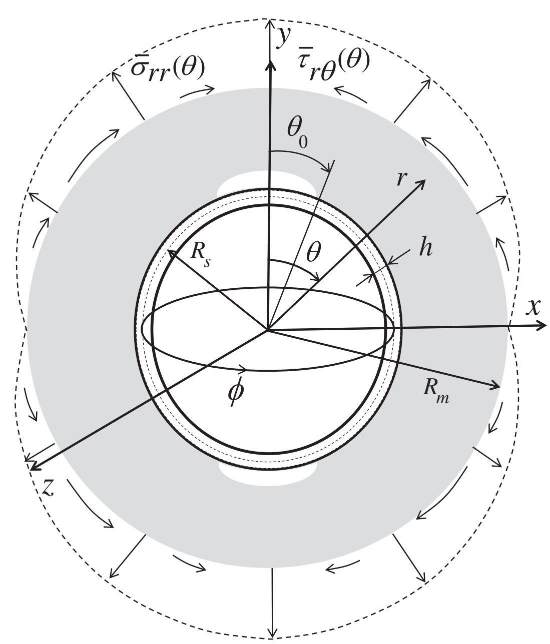

Fig. 1. Schematic depiction of the system geometry and nomenclature.