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Peristaltic pumping of a viscoelastic fluid at high occlusion ratios and large Weissenberg numbers

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ABSTRACT

Peristaltic pumping is a mechanism for transporting fluid or immersed particles in a channel by waves of contraction. It occurs in many biological organisms as well as in several human designed systems. In this study, we investigate numerically the peristaltic pumping of an incompressible viscoelastic fluid using the simple Oldroyd-B model coupled to the Navier–Stokes equations. The pump's walls are assumed to be massless immersed fibers whose prescribed periodic motion and flow interaction is handled with the Immersed Boundary Method. We utilize a new, highly efficient non-stiff version of this method which allows us to explore an unprecedented range of parameter regimes, nearly all possible occlusion ratios and Weissenberg numbers in excess of 100. Our numerical investigation reveals rich, highly concentrated stress structures and new, striking dynamics. The investigation also points to the limitations of the Oldroyd B model, with a potential finite time blow-up, and to the role of numerical regularization.

1. Introduction

Peristalsis is the flow that takes places in a channel with flexible walls due to a series of contraction or expansion waves along the walls. It is a predominant mechanism of action in a variety of biological phenomena, from earthworm mobility [28] to gastrointestinal [15] and esophageal transport [2]. Peristalsis is utilized in many mechanical fluid pumps, often because of its ability to effectively transport highly viscous fluids and as well as immersed particles. In both biological and mechanical systems, the fluid internal to the pump may be non-Newtonian. Such is the case for peristalsis in the oviduct [6] and uterus [19] where the transported biological fluid is highly viscoelastic [18].

There are several analytical and numerical studies of peristalsis [30,17,16,7,27,32,14,35,12,11,39] with increasing emphasis on viscoelastic fluids. We focus here on a model used by the recent investigations of Teran, Fauci, and Shelley [35] and by Chrispell and Fauci [11]. The fluid model is based on the Stokes [35] or Navier–Stokes [11] equations coupled with the simple Oldroyd-B (OB) model and describes a dilute solution of flexible polymeric molecules represented by Hookean dumbbells [5,13,20]. The Immersed Boundary (IB) Method [24,25] is employed to model the pump, as done originally by Fauci [14]. In this IB setting, the pump's walls are tethered to anchor points which are set to a prescribed periodic motion to simulate the waves of contraction and expansion. Teran, Fauci, and Shelley [35] and Chrispell and Fauci [11] found that there is a marked difference between the Newtonian and the non-Newtonian fluid pumping. In particular, the mean flow rate is noticeably affected by viscoelastic effects. They also noted that extremely strong normal stresses are generated at the pump's constriction as the amplitude of the peristaltic wave relative to the channel width (the so called occlusion ratio) increases, even for moderate Weissenberg numbers (the polymer relaxation time relative to the flow's characteristic time scale). These large normal stresses present a formidable computational challenge; to accurately preserve the structure of the pump's walls during their prescribed periodic motion very stiff boundary forces must be employed. This induces a severe time step restriction for explicit IB methods [34,33]. Indeed, the use of an explicit IB method in [35,11] limited the parameter space ameanable to simulation to a region consisting of only the first half of the possible occlusion ratios and to Weissenberg numbers less or equal to 5. In this work, we employ a novel semi-implicit IB method [9,8] to make possible a computational study that covers nearly all possible occlusion ratios and Weissenberg numbers in excess of 100. Our numerical investigation reveals new, striking dynamics which include highly localized stress structures, a potential finite time blow-up, symmetry breaking transitions, and the emergence of a critical occlusion ratio at which the ordering of the mean flow rate with respect to the Weissenberg number is reversed.

The rest of this article is organized as follows. First, the model is described in detail in the following section. Then, in Section 3, we present a new numerical method coupling a viscoelastic fluid solver to a novel, highly efficient semi-implicit version of the IB method,

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[9,8]. Our numerical results are presented and discussed in Section 4. Finally, some concluding remarks are given in Section 5.

2. The peristaltic pump and viscoelastic fluid models

We consider a peristaltic pump immersed in a periodic 2D domain $\Omega = [0,1] \times [0,1]$. We model the peristaltic pump's walls as two disconnected sinusoidal curves [14,35,11]

$$\mathbf{X}(t) = \left\{ \left(\xi, \frac{1}{2} + d(\xi,t) \right) \Big| \xi \in [0,1] \right\} \cup \left\{ \left(\xi, \frac{1}{2} - d(\xi,t) \right) \Big| \xi \in [0,1] \right\}, \quad (1)$$

where

$$d(x,t) = \frac{\alpha}{2\pi}[1 + \chi \sin 2\pi(\xi - t)]. \quad (2)$$

Both the spatial and temporal period of the pump is fixed at 1. As time progresses, the waves of peristalsis move from left to right, forcing the fluid to flow to the right (in aggregate). The parameter χ represents the occlusion ratio of the pump. The value $\chi = 0$ corresponds to a straight channel, with no waves of peristalsis, while $\chi = 1$ correspond to a completely occluded channel with the peaks of each sinusoidal curve meeting at some point along the horizontal line $y = 1/2$. The parameter α controls the aspect ratio of the channel. For this work we fix $\alpha = 1.5$.

We model the interior and exterior of the valve as a dilute, incompressible OB suspension [5]. The interaction between the valve and the fluid is captured via the IB Method. The continuous equations are then

$$Re\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u}\right) = -\nabla p + \nabla^2 \mathbf{u} + \beta \nabla \cdot \mathbf{S} + \mathbf{f}, \quad (3)$$

$$\nabla \cdot \mathbf{u} = 0, \quad (4)$$

$$\frac{\partial \mathbf{X}}{\partial t} = \mathbf{u}(\mathbf{X}, t), \quad (5)$$

$$\mathbf{S}^{\nabla} = -We^{-1}(\mathbf{S} - \mathbf{I}). \quad (6)$$

Here $\mathbf{f}$ is a very stiff force acting on the immersed walls binding the current configuration $\mathbf{X}$ to the desired prescribed position given by (1). Because of this specified motion, the fluid-immersed boundary interaction is only one-way. In (3), Re is the Reynolds number which is a measure of the viscous dissipation relative to inertial forces. The dimensionless term β specifies the strength of the viscoelastic force $\nabla \cdot \mathbf{S}$. Here, $\mathbf{S}$ is the deviatoric part of the viscoelastic stress tensor and evolves according to the OB constitutive equation, given in (6) [5]. $\mathbf{S}^{\nabla}$ denotes the upper convected derivative of $\mathbf{S}$:

$$\mathbf{S}^{\nabla} = \frac{d\mathbf{S}}{dt} + \mathbf{u} \cdot \nabla \mathbf{S} - \nabla \mathbf{u} \cdot \mathbf{S} - \mathbf{S} \cdot \nabla \mathbf{u}^T. \quad (7)$$

We is the Weissenberg number and is defined as the ratio of the polymer relaxation time τ_p and the characteristic time scale of the flow $\tau_f = L/V$, where V is the speed of the contractile wave and L is its wavelength. Alternatively, a Deborah number De could be defined by $De = \tau_p/\tau_f$ with $\tau_f = R/\langle v \rangle$ where R is channel's constriction width and $\langle v \rangle$ is the fluid average velocity [4]. Note that in the limit as $We \to 0$ the polymeric stress is fixed as the identity tensor $\mathbf{I}$ and the fluid becomes Newtonian.

Finally, the product βWe can be interpreted as the ratio of the polymeric viscosity to the solvent viscosity [35]. We fix $\beta We = \frac{1}{2}$ following [35]. This particular choice is motivated by the experiments in [1] where the solution's viscosity ratio is 1/2. We choose the characteristic length scale to be 1, the width of our fluid domain Ω and the characteristic time scale we also take it to be 1, the period of the peristaltic pump. We fix the Reynolds number of our fluid at $Re = 1$ as in [11]. We have looked at lower Re and found that for $Re \leqslant 1$ inertial effects are negligible. Throughout this work the only *fluid* parameter we change is the Weissenberg number We.

We discretize the pump's walls $\mathbf{X}$ as a collection of N_B immersed points $\{\mathbf{X}_j\}$. The position of these points is not directly prescribed, rather we construct an artificial force to approximately constrain the immersed points to their respective positions. For each point $\mathbf{X}_j$, we define $\mathbf{T}_j$ to be the desired target position. We then induce a force $\mathbf{F}$ on immersed points given by

$$\mathbf{F} = \sigma(\mathbf{T} - \mathbf{X}). \quad (8)$$

The stiffness coefficient σ is a numerical parameter. In the limit as $\sigma \to \infty$ we exactly constrain $\mathbf{X}$ to the desired configuration. In practice, σ needs to be a fairly large value. With our semi-implicit method we can use values of σ multiple orders of magnitude larger than previously possible. For the large values of χ and We explored in this work, we are required to take $\sigma = O(10^6)$ to maintain the structure of the pump. This large stiffness coefficient would lead to prohibitively small time-steps for explicit methods. In our numerical experiments our choice of σ reduces deviations in $\mathbf{X}$ from the target position $\mathbf{T}$ to less than 0.0005 units, even when the normal polymeric stresses at the pump's walls rise to values of 1000 and more.

3. Numerical methodology

We briefly overview the numerical method here. It is based on a semi-implicit discretization of the Navier–Stokes equations given by

$$\frac{\mathbf{u}^{n+1} - \mathbf{u}^n}{\Delta t} + \mathbf{u}^n \cdot \nabla \mathbf{u}^n = -\mathbf{D}_h p^{n+1} + L_h \mathbf{u}^{n+1} + \mathbf{f}, \quad (9)$$

$$\mathbf{D}_h \cdot \mathbf{u}^{n+1} = 0, \quad (10)$$

$$\frac{\mathbf{X}^{n+1} - \mathbf{X}^n}{\Delta t} = \mathcal{S}_n^* \mathbf{u}^{n+1}. \quad (11)$$

Here a superscript n denotes a numerical approximation taken at the time $n\Delta t$ and Δt is the timestep. The spatial operators $\mathbf{D}_h$ and L_h are the standard, second order approximations to the gradient and the Laplacian, respectively. The convection term $\mathbf{u}^n \cdot \nabla \mathbf{u}^n$ is handled separately via a third-order essentially non-oscillatory (ENO) scheme [31]. The force $\mathbf{F}$ in (8) is defined at the immersed boundary only and has to be spread onto the surrounding Eulerian grid points. Likewise, the velocity field is not given at the immersed boundary, so we must *interpolate*. To achieve this spreading and interpolation we define the adjoint operators:

$$(\mathcal{S}_n G)(\mathbf{x}) = \sum_{s \in \mathcal{G}_B} G(s) \delta_h(\mathbf{x} - \mathbf{X}^n(s)) h_B, \quad (12)$$

$$(\mathcal{S}_n^* w)(s) = \sum_{\mathbf{x} \in \mathcal{G}_\Omega} w(\mathbf{x}) \delta_h(\mathbf{x} - \mathbf{X}^n(s)) h^2, \quad (13)$$

known as the spreading and interpolation operators, respectively. Here $\delta_h(\mathbf{x}) \equiv d_h(x) d_h(y)$ is an approximation to the two-dimensional Dirac delta distribution and d_h is given by

$$d_h(r) = \begin{cases} \frac{1}{4h}\left(1 + \cos\left(\frac{\pi r}{2h}\right)\right) & \text{if} |r| \leqslant 2h, \\ 0 & \text{otherwise}. \end{cases} \quad (14)$$

We refer to these operators as lagged because the interface configuration $\mathbf{X}^n$ is used instead of the future configuration $\mathbf{X}^{n+1}$.

Utilizing $\mathcal{S}_n$ and $\mathcal{S}_n^*$ we now specify the form of $\mathbf{f}$ in (9):

$$\mathbf{f} = \sigma \mathcal{S}_n(\mathbf{T}^{n+1} - \mathbf{X}^{n+1}) + \beta \mathbf{D}_h \cdot \mathbf{S}^n, \quad (15)$$

which incorporates both the artificial force on the immersed points, as well as the additional force coming from the polymeric stress. We thus consider the polymeric stress fixed as we update the fluid. Once we have an updated fluid velocity $\mathbf{U}^{n+1}$ we will then calculate an updated value for the stress $\mathbf{S}^{n+1}$.