Composite Structures 109 (2014) 86–92

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

Modeling of corrugated laminates

M. Winkler *, G. Kress

Centre of Structure Technologies, ETH Zürich, Leonhardstr. 27, CH-8092 Zürich, Switzerland

ARTICLE INFO

Article history:
Available online 1 November 2013

Keywords:
Corrugated laminates
Anisotropy
Modeling
Finite element analysis
Shell theory

ABSTRACT

This work compares two modeling approaches for corrugated laminates. Both models use a unit cell and a generalized plane-strain approach in order to determine a substitute plate model. The first model is based on a linear thin-shell theory and the second model uses a self-programmed finite element program with planar elements. The comparisons show the differences of the models as well as the limits of the shell theory approach. Furthermore, the realizable anisotropy of corrugated laminates which is important for morphing wing applications is investigated.

1. Introduction

In the context of developing new aircraft, morphing wings are a current research topic at several research institutions. In order to realize a morphing wing the usage of a flexible skin is required. Thill et al. [1] summarized in a detailed literature review different design possibilities for such flexible skins. Gandhi and Anusonti–Inthra [2] made some studies on the requirements of the skin. Both investigations showed that a highly anisotropic structure has to be used, as the skin has to be compliant in the chordwise direction and stiff in the spanwise direction.

A possible design for the skin is the usage of corrugated laminates. The corrugations increase the stiffness in one direction, whereas the stiffness decreases in the perpendicular direction. Experimental work on corrugated laminates and the derivation of simple relations for extensional and bending stiffnesses were done by Yokozeki et al. [3]. Thill et al. focused on the application of corrugated laminates to the trailing edge area of a wing. An overview over the performed investigations can be found in [4,5]. On the one hand there were conducted several experimental studies on the mechanical properties of corrugated laminates which included comparisons to simple formulas for extensional and bending stiffnesses [6,7]. These studies dealt with the consideration of influence parameters like corrugation geometry, laminate thickness or material. A more detailed analysis based on nonlinear finite element calculations was used to explain the exceptional behavior of trapezoidal aramid/epoxy laminates undergoing large displacements [8]. On the other hand also the aerodynamics was investigated by wind tunnel tests and computational fluid dynamics calculations [9–11]. These studies showed that a suitable choice of corrugation shape, period length, amplitude and Reynolds number can decisively reduce the disadvantages of the non-smooth surface. Xia et al. investigated the aerodynamic performance of corrugated skins – where most part of the considered standard wing profile was covered by a corrugated structure – by numerical and experimental methods [12].

Two complete models for corrugated laminates that are considering a substitute plate model with all relevant load cases are existing [13–16]. The first model deals with an analytical thin-shell theory approach with closed-form solutions for the substitute stiffnesses and strain limits for symmetric and balanced cross-ply laminates [13,14,16]. Due to the used linear thin-shell theory aspects like geometrical nonlinearities or influence of thick laminates cannot be considered. The second approach [15,16] uses a plane finite element (FE) which was derived on the assumption of a generalized plane-strain state. This model also allows the complete calculation of the substitute stiffness matrix. A general laminate with a general corrugation geometry can be considered. Investigations on the influence of the corrugation geometry can be found in [16,17]. Xia and Friswell [18,19] recently presented a model which allows to calculate the stiffnesses of the main diagonal. But they used several simplifications like the neglect of the Poisson ratio. Besides these models, there are existing several models based on orthotropic plate models [20–26]. However, these models are built for isotropic materials.

In our previous work [15] discrepancies for the coupling stiffnesses between the analytical model and the FE approach were revealed for certain fiber orientations of unidirectional laminates. The differences between the two models lead to the present work. The main modeling ideas will be presented in an abbreviated form within Section 2. A comprehensive description of all modeling details of both models can be found in [16]. A comparison of the two

* Corresponding author. Address: ZF Friedrichshafen AG, Graf-von-Soden-Platz 1, 88046 Friedrichshafen, Germany. Tel.: +49 7541 77 4865.
E-mail address: michael.winkler@alumni.ethz.ch (M. Winkler).

0263-8223/$ - see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.compstruct.2013.10.048

models will then be shown within Section 3. The focus lies on the influence of the thickness-to-radius ratio and on occurring deviations between both models in this context. Section 4 deals with the realizable anisotropy depending on geometry and on material anisotropy of the base material. This will deliver insight into the potential anisotropy range that can be used for applications such as morphing skins.

2. Modeling approach

As a corrugated laminate is a periodic structure with many repeating sections, it is advantageous to analyze only one representative unit cell which can be seen in Fig. 1. This approach is valid as long as the considered structure is large enough so that edge effects can be neglected. Thereby, the usage of a unit cell has two main advantages. Firstly, such an approach allows to investigate the local behavior of displacements, strains or stresses. A rather small model is then sufficient for such investigations. Secondly, the unit cell can be used to homogenize the corrugated structure so that it can be represented by means of a substitute plate model. This helps to reduce the computational effort for simulations of large structures.

Fig. 1 illustrates the parameterization of the unit cell and the coordinate system definition. The considered corrugated structure consists of two circular segments. In general, also other corrugation patterns like trapezoidal or triangular ones are possible. The unit cell is infinitely long in x direction, as edge effects are neglected. In y direction the unit cell comprises of one period length P. The circular geometry of the unit cell is dependent on the period length P and the half-amplitude c. These two parameters then give the radius R and the angle ψ_0 which will also be used for a more concise notation in the following sections.

2.1. Generalized plane-strain approach

Due to the mentioned simplification with respect to the x direction, the strains and stresses are not a function of x. Whereas, the displacements can be a function of x. This is also known as a generalized plane-strain state [27]. The general equilibrium equations in cartesian coordinates can accordingly be simplified to:

$$\begin{aligned} \tau_{yx,y} + \tau_{zx,z} &= 0 \\ \sigma_{y,y} + \tau_{zy,z} &= 0 \\ \tau_{yz,y} + \sigma_{z,z} &= 0 \end{aligned} \tag{1}$$

The displacement field in cartesian coordinates of Eq. (2) is compatible with the equilibrium equations.

$$\begin{aligned} u_x &= u_x(y,z) + x\hat{\epsilon}^0_x + zx\hat{\epsilon}^1_x + \frac{1}{2}yz\hat{\epsilon}^1_{xy} \\ u_y &= u_y(y,z) + \frac{1}{2}xz\hat{\epsilon}^1_{xy} \\ u_z &= u_z(y,z) - \frac{1}{2}x^2\hat{\epsilon}^1_x - \frac{1}{2}xy\hat{\epsilon}^1_{xy} \end{aligned} \tag{2}$$

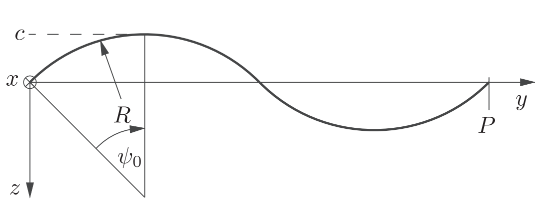

Fig. 1. Parameterization of the unit cell (adapted from [16]).

Thereby, the terms containing $\hat{\epsilon}^j_i$ represent prescribed deformations. These comprise uniform strain $\hat{\epsilon}^0_x$ in x direction, uniform bending $\hat{\epsilon}^1_x$ as well as twist $\hat{\epsilon}^1_{xy}$ about the y axis. With the linear kinematical relations the strains in cartesian coordinates can be determined:

$$\begin{aligned} \epsilon_x &= u_{x,x} = \hat{\epsilon}^0_x + z\hat{\epsilon}^1_x \\ \epsilon_y &= u_{y,y} \\ \epsilon_z &= u_{z,z} \\ \gamma_{yz} &= u_{y,z} + u_{z,y} \\ \gamma_{xz} &= u_{x,z} \\ \gamma_{xy} &= u_{x,y} + z\hat{\epsilon}^1_{xy} \end{aligned} \tag{3}$$

2.2. Modeling with substitute plates

The already mentioned homogenization of corrugated laminates can be made with the help of a general substitute plate similar to classical lamination theory [28] which is usually used for flat laminated plates:

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} \widetilde{A}_{11} & \widetilde{A}_{12} & \widetilde{A}_{16} & \widetilde{B}_{11} & \widetilde{B}_{12} & \widetilde{B}_{16} \\ \widetilde{A}_{12} & \widetilde{A}_{22} & \widetilde{A}_{26} & \widetilde{B}_{12} & \widetilde{B}_{22} & \widetilde{B}_{26} \\ \widetilde{A}_{16} & \widetilde{A}_{26} & \widetilde{A}_{66} & \widetilde{B}_{16} & \widetilde{B}_{26} & \widetilde{B}_{66} \\ \widetilde{B}_{11} & \widetilde{B}_{12} & \widetilde{B}_{16} & \widetilde{D}_{11} & \widetilde{D}_{12} & \widetilde{D}_{16} \\ \widetilde{B}_{12} & \widetilde{B}_{22} & \widetilde{B}_{26} & \widetilde{D}_{12} & \widetilde{D}_{22} & \widetilde{D}_{26} \\ \widetilde{B}_{16} & \widetilde{B}_{26} & \widetilde{B}_{66} & \widetilde{D}_{16} & \widetilde{D}_{26} & \widetilde{D}_{66} \end{bmatrix} \begin{Bmatrix} \hat{\epsilon}^0_x \\ \hat{\epsilon}^0_y \\ \hat{\epsilon}^0_{xy} \\ \hat{\epsilon}^1_x \\ \hat{\epsilon}^1_y \\ \hat{\epsilon}^1_{xy} \end{Bmatrix} \tag{4}$$

where N_i are line loads and where M_i are line moments. These can be determined by the product of the $\widetilde{\mathbf{A}}\widetilde{\mathbf{B}}\widetilde{\mathbf{D}}$ substitute stiffness matrix and the according prescribed membrane strains $\hat{\epsilon}^0_i$ and curvatures $\hat{\epsilon}^1_i$. The tilde indicates that the $\widetilde{\mathbf{A}}\widetilde{\mathbf{B}}\widetilde{\mathbf{D}}$ matrix represents the homogenized behavior of the corrugated laminate. If the half-amplitude c is equal to zero, the stiffness matrices **ABD** and $\widetilde{\mathbf{A}}\widetilde{\mathbf{B}}\widetilde{\mathbf{D}}$ coincide.

Six load cases are necessary in order to determine the entries of the substitute stiffness matrix. Fig. 2 illustrates the load cases. Thereby, the undeformed unit cell is colored gray and the deformed structure is indicated by black solid lines.

For each load case only the respective prescribed membrane strain or curvature is unequal to zero. The load cases one and two elongate the unit cell in x direction with the membrane strain $\hat{\epsilon}^0_x$ and in y direction with the membrane strain $\hat{\epsilon}^0_y$, respectively. The shear strain $\hat{\epsilon}^0_{xy}$ is applied for load case three. The load cases four and five bend the unit cell about the y axis by the constant curvature $\hat{\epsilon}^1_x$ and about the x axis by the curvature $\hat{\epsilon}^1_y$, respectively. Finally, the unit cell is twisted by the curvature $\hat{\epsilon}^1_{xy}$ about the y axis for load case six. Additionally to these boundary conditions, periodicity conditions have to be considered due to the unit cell approach. Thus, both ends of the unit cell are subjected to the same deformations. For the analytical shell theory approach also continuity conditions between the two circular halves of the unit cell have to be used, as the unit cell consists of two circular segments.

By dividing the reactions $N_x, N_y, N_{xy}, M_x, M_y$ and M_{xy} by the according prescribed membrane strain or curvature the substitute stiffness matrix entries can be determined:

$$\begin{aligned} \widetilde{A}_{ij} &= \frac{N_j}{\hat{\epsilon}^0_i}, \qquad i,j = 1,2,6 \text{ or } x,y,xy \\ \widetilde{B}_{ij} &= \frac{M_j}{\hat{\epsilon}^0_i}, \qquad i,j = 1,2,6 \text{ or } x,y,xy \end{aligned} \tag{5}$$