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A fully nonlinear dynamic formulation for rotating composite beams: Nonlinear normal modes in flapping

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ABSTRACT

The geometrically exact equations of motion of pretwisted rotating composite beams parametrized by one space coordinate are derived from three-dimensional theory. The mechanical formulation is based on the special Cosserat theory of rods which does not place any restriction on the geometry of deformation besides the rigidity of the cross-sections. The constitutive relations for the composite beams are obtained within the context of three-dimensional theory. The Taylor expansion of the equations of motion – further specialized to the case of unshearability – is carried out about the equilibrium caused by centrifugal forces to obtain the linearized and third-order perturbed forms of the governing equations. The Galerkin approach is employed to project the linearized equations of motion and solve for the linear free vibration characteristics. Subsequently, the method of multiple scales is applied directly to the perturbed equations of motion to investigate the nonlinear flapping modes and their frequency variations with the amplitude and the angular speed. These modes are investigated away from internal resonance conditions found as the critical angular speeds for which nonlinear interactions between different modes may occur. Results obtained by the Galerkin method are compared with those obtained via a finite element discretization.

1. Introduction

The study of the dynamic behavior of slender rotating beams, frequently employed in a variety of rotating machineries such as helicopters and turbines, is a major step of the overall design process. Due to the inherently poor out-of-plane stiffness of laminated composite structures and the significant slenderness of long-span blades, the derivation of a one-dimensional geometrically exact dynamic model of composite rotating beams allows to perform parametric stability analysis enabling efficient design, control and structural health monitoring of such systems.

The field of dynamic modeling of isotropic rotating beams is much richer than that of composite beams. Hodges and Dowell [1] used Hamilton's principle and the Newtonian method for obtaining the equations of motion of a long, straight rotating asymmetric, isotropic blade with a variable pretwist angle and a small precone angle. They employed the exact strain–displacement relationships, however, they dealt with moderate deflections.

Crespo da Silva and Hodges [2] derived the equations of motion of a typical rotating blade with a precone angle and a variable pitch angle including the effects of higher order nonlinearities and aerodynamic forces using Hamilton's principle. Later Crespo da Silva [3] obtained the fully nonlinear equations of motion of the rotating blade including the geometric nonlinearities. He derived the equilibrium solution and the eigenvalues and eigenvectors of the perturbed system under aerodynamic forces and investigated the stability of the perturbed blade about its equilibrium.

Lacarbonara et al. [4] obtained the equations of motion of linearly elastic, isotropic blades with nonsymmetric cross-sections via a geometrically exact approach. By applying the Galerkin method to the study of linear free vibration of unshearable blades including couplings between flapping, lagging, axial and torsional modes, they obtained necessary conditions for the onset of different types of internal resonances depending on the angular speed.

A few works have dealt with nonlinear free vibrations of rotating beams. Avramov et al. [5] derived the nonlinear flexural-flexural–torsional coupled equations of motion of rotating asymmetric blades. They computed the backbone curves of the nonlinear modes and predicted a softening behavior for the first and fourth modes and a hardening behavior for the second and third modes, respectively. Turhan and Bulut [6] investigated vibrations of a rotating Euler–Bernoulli beam by applying the Lindstedt-Poincaré method and the method of multiple scales to the discretized integro-partial differential equations of motion

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with third-order curvature terms. They studied the effects of angular speed on nonlinear dynamics of rotating beams. They also presented results on hardening or softening nonlinearity and jump phenomena under external periodic excitation.

Valverde and Garcia-Vallejo [7] implemented the absolute nodal coordinate formulation to study the stability of a rotating beam. They compared their results with those obtained using the Kirchhoff–Love model.

Arvin and Bakhtiari-Nejad [8] applied the method of multiple scales to the discretized equations of motion of a rotating Euler–Bernoulli beam obtained by Hamilton's principle. They constructed the individual nonlinear normal modes (NNMs) or the coupled NNMs under internal resonance conditions. They investigated the stability and bifurcations of NNMs in three-to-one and two-to-one internal resonances, respectively, between two flapping modes and between a flapping mode and an axial mode.

Arvin and Lacarbonara [9] applied the direct method of multiple scales to the third-order perturbed form of the equations of motion [4] to construct the flapping NNMs of rotating isotropic beams. They also analyzed the effects of the angular speed on the nonlinear flapping frequency variation with the oscillation amplitude.

Besides these studies on rotating elastic, isotropic blades, a few authors have studied an exact formulation of rotating composite blades. Hodges [10] derived the nonlinear intrinsic formulation for the dynamics of rotating pre-curved and twisted anisotropic beams for which the effects of warping were considered. A mixed approach combining the Newtonian and the variational approach was employed to obtain the equations of motion.

Pai and Nayfeh [11] derived the nonlinear equations of motion for pre-curved and twisted composite rotor blades undergoing large displacements. They used the coordinate transformation, and extended Hamilton's principle to derive six nonlinear equations of motion including the axial, bending, torsional and shearing modes. The formulation was based on an energy approach, while the derivation of the equations of motion was based on the Newtonian approach. They included the in-plane and out-of-plane warping effects due to bending, extensional, shearing and torsional loadings, elastic couplings between the warping functions, and 3D stress effects by using the results of a static, sectional, finite element analysis.

Hodges [12] formulated exact nonlinear equations of motion for initially curved and twisted anisotropic beams. By the Kirchhoff analogy, he showed that a time-discretization scheme for nonlinear dynamics of rigid bodies can be applied in the same manner to space discretization of the nonlinear equilibrium problem.

Hsu [13] derived the nonlinear equations of motion of pretwisted orthotropic composite rotor blades using the von Karman strain–displacement relationships via Hamilton's principle. He analyzed the effects of fiber orientation and damping on the nonlinear response of the rotating blade under external forcing using the differential quadrature method.

Saravia et al. [14] investigated the dynamic stability behavior of thin-walled rotating composite beams including anisotropy, shear flexibility, warping deformations and rotary inertia using the finite element method. They studied the effects of angular speed as well as those associated with the laminate fibers angles on the natural frequencies and the regions of stability. Machado and Saravia [15] examined the nonlinear principal parametric resonance response of a cantilever rotating slender Euler–Bernoulli beam subject to a harmonic transverse load. By considering the von Karman strain–displacement relationships, they obtained the equations of motion in the form of an integro-partial differential equation. They used the direct method of multiple scales to determine the steady-state response and the system stability.

Arvin and Bakhtiari-Nejad [16] derived the equations of motion of rotating composite, shear deformable beams featuring the von Karman strain–displacement relationships via Hamilton's principle. The initial equilibrium due to the centrifugal forces was found by the differential transform method. The Galerkin discretization scheme was employed to find the linear free vibration characteristics. The dependence of the backbone curves of the lowest three flapping modes on the angular speed and the lamination scheme was studied therein by applying the direct method of multiple scales. Moreover, in [17] they investigated the stability of the flapping nonlinear modes in rotating composite beams with internal resonances.

Linear free vibrations of composite rotating blades were investigated in a few works. Chandiramani et al. [18] investigated linear free vibration behavior of rotating laminated composite thick/thin-walled box beams using a modified Galerkin method. They considered the anisotropy, transverse shear flexibility, warping inhibition, centrifugal and Coriolis forces effects, however they discarded the Coriolis-induced effects during the solution process.

Lee et al. [19] investigated the flap-wise bending motion by considering a cantilever beam with small thickness/width ratio and symmetric layup. They examined the effects of angular speed and fiber orientation on the natural frequencies. Yoo et al. [20] extended their previous work [19] by considering shearable blades. They eliminated the coupling effect between the two bending motions, by assuming a set of skew-symmetric fiber angles for the layers, while they considered the coupling effects between axial and flap-wise bending motions. They applied Kane's method together with the Rayleigh–Ritz assumed-mode method to derive the equations of motion. They investigated the hub radius and fiber angles effects on the natural frequencies.

Choi et al. [21] studied the bending vibration control of a pretwisted rotating composite thin-walled beam. The formulation was based on single-cell composite beam including a warping function, centrifugal and Coriolis forces, pretwist angle and piezoelectric effects.

In this paper, the one-dimensional geometrically exact equations of motion for pretwisted composite multilayer beams are derived from three-dimensional elasticity. The exact nonlinear equations of motion are expanded in Taylor series to obtain the perturbed form of the equations of motion about the initial equilibrium up to third order. The Galerkin discretization approach is applied to the partial differential equations of motion to obtain a system of ODEs allowing to unfold the linear free vibration features of unshearable blades. In particular, the critical speeds for which internal resonances between different modes may occur are considered. The method of multiple scales is applied to the third-order perturbed form of the equations of motion to investigate the nonlinear frequency variation of the flapping modes by varying the angular speed. It is ascertained that at high angular speeds the increase of frequencies of the flapping modes due to the centrifugally-induced stiffness is such to possibly activate two-to-one internal resonances between flapping and axial modes (i.e., an autoparametric interaction mechanism), situation that can profoundly affect the overall dynamics.

2. Dynamical formulation

First, a brief review of the kinematic model and the equations of motion is addressed. The kinematics and the balance equations are the same as those employed for elastic isotropic blades in [4]. The governing equations for composite blades differ from those of isotropic blades for the constitutive laws presented in the subsequent section.

In this paper, Gibbs notation is adopted for vectors and tensors. Euclidean vectors and vector-valued functions are denoted by lower-case, italic, bold-face symbols. The dot product and cross-product of (vectors) $\boldsymbol{u}$ and $\boldsymbol{v}$ are denoted by $\boldsymbol{u} \cdot \boldsymbol{v}$ and $\boldsymbol{u} \times \boldsymbol{v}$,