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# Nonlinear stability analysis of imperfect three-phase polymer composite plates in thermal environments

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ABSTRACT

This paper presents an analytical investigation on the nonlinear response of the thin imperfect laminated three-phase polymer composite plate in thermal environments. The formulations are based on the classical plate theory taking into account the interaction between the matrix and the particles, geometrical nonlinearity, initial geometrical imperfection. By applying Galerkin method, explicit relations of load–deflection curves are determined. Obtained results show effects of the fibers and the particles, material, geometrical properties and temperature on the buckling and post-buckling loading capacity of the three phase composite plate, therefore we can proactively design materials and structural composite meet the technical requirements as desired when adjustment components.



## 1. Introduction

Three phase composite is a material consisted of matrix of the reinforced fibers and particles which have been investigated by Vanin and Duc since 1996. They have determined the elastic modulus for three phases composite 3Dm [1] and 4Dm [2]. Their findings have shown that the fibers are able to improve the elastic modulus, the particles can resist to the penetration and the heat, reduce the creep deformations and the defects in materials.

Despite of a large number of applications, our understanding on the structure of three phase composite materials (plate and shell) is not much. The general view of three-phase composite can be found in [3]. Recently, there are several claims on the deflection and the creep for the three phase composite plate in the bending state [4]. These findings have shown that optimal three-phase composite can be obtained by controlling the volume ratios of fiber and particles.

Plate, shell and panel are basic structures used in engineering and industry. These structures play an important role as main supporting component in all kind of structure in machinery, civil engineering, ship building, flight vehicle manufacturing, etc. The stability of composite plate and shell is the first and most important problem in optimal design. In fact, many researchers are interested in this problem including the studies of the composite plates [4–12]. However, researches on the stability of three-phases composite plates and shells are very few. Whereas, the choice of a suitable ratio of components materials in three-phases composite is very important in designing new composite materials and predict mechanical and physical properties of advanced designed materials. Therefore, from scientific and practical point of view, it is, therefore, very important and meaningful to carry an investigation on the three phase composite plate and shell. Actively choosing material components and ratio of its mixing allows us to decide the advance materials and forecasting its physic-mechanical characteristics.

Several fundamental references on composite plates and shells are Brush and Almroth [13], Reddy [14] (for laminated composite plate and shell) and Shen [15] (for composite FGM). Some research on the stability of laminated composite and FGM plates can be obtained in [8–12].

Recently, in [7] we have studied nonlinear stability of the three-phase polymer composite plate under mechanical loads. In the present paper, we have studied the nonlinear stability of three-phase polymer composite with imperfections in thermal environments. The paper focuses on deriving the algorithm for calculating the stability of three-phase composite by analyzing the load–deflection relationship base on the basic equations of laminated composite, while also studies the effect of component material properties, temperature, geometrical properties and imperfection on the stability of three-phase polymer composite plate.

A special point of the results is to show the algorithms explicit determine the coefficients of thermal expansion of the three-phase composite material on the elastic modulus, coefficients of thermal expansion and the ratio of component material (the elastic modules of three-phase composite material was determined by theoretical and experimental methods are published in [7,16]). Moreover, the first time the article showed performances Hooke's law relationship of stress–strain three-phase composite plate include the effects of temperature. Thereby, we can calculate nonlin-

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ear behavior of three-phase composite plates under temperature loads and determine the effect of the component elements and structure of materials on thermal stability of the plate.

## 2. Determine the elastic modules and the effective thermal expansion coefficients of three-phase composite

### *2.1. Determine the elastic modules of composite*

The elastic modules of three-phase composites are estimated using two theoretical models of the two-phase composite consecutively: $n\mathrm{D_m} = \mathrm{O_m} + n\mathrm{D}$ [1,2,7]. This paper considers three-phase composite reinforced with particles and unidirectional fibers, so the problem's model will be: $1\mathrm{D_m} = \mathrm{O_m} + 1\mathrm{D}$. Firstly, the modules of the effective matrix $\mathrm{O_m}$ which called "effective modules" are calculated. In this step, the effective matrix consists of the original matrix and particles, it is considered to be homogeneous, isotropic and have two elastic modules. The next step is estimating the elastic modules for a composite material consists of the effective matrix and unidirectional reinforced fibers.

Assume that all the component phases (matrix, fiber and particle) are homogeneous and isotropic, we will use $E_m$, $E_a$,$E_c$; $\nu_m$, $\nu_a$, $\nu_c$; $\psi_m$, $\psi_a$; $\psi_c$ to denote Young modulus and Poisson ratio and volume ratio for matrix, fiber and particle, respectively. According to Vanin and Duc in [1,2], we can obtain the modules for the effective composite as

$$\overline{G} = G_m\frac{1-\psi_c(7-5\nu_m)H}{1+\psi_c(8-10\nu_m)H}, \tag{1}$$

$$\overline{K} = K_m\frac{1+4\psi_c G_m L(3K_m)^{-1}}{1-4\psi_c G_m L(3K_m)^{-1}}, \tag{2}$$

here

$$L = \frac{K_c - K_m}{K_c + \frac{4G_m}{3}}, \quad H = \frac{G_m/G_c - 1}{8-10\nu_m + (7-5\nu_m)\frac{G_m}{G_c}}. \tag{3}$$

$\overline{E}, \bar{\nu}$ can be calculate from $(\overline{G}, \overline{K})$ as

$$\overline{E} = \frac{9\overline{K}\overline{G}}{3\overline{K}+\overline{G}}, \quad \bar{\nu} = \frac{3\overline{K}-2\overline{G}}{6\overline{K}-2\overline{G}}. \tag{4}$$

We should note that formulas (1) and (2) take into account the nonlinear effects and the interaction between the particles and the base. These are different from the other well-known formulas.

The elastic modules for 3-phase composite reinforced with unidirectional fiber are chosen to be calculated using Vanin's formulas [17] with six independent elastic modulus

$$\begin{aligned}
E_1 &= \psi_a E_a + (1-\psi_a)\overline{E} + \frac{8\overline{G}\psi_a(1-\psi_a)(\nu_a-\bar{\nu})}{2-\psi_a+\bar{x}\psi_a+(1-\psi_a)(x_a-1)\frac{\overline{G}}{G_a}},\\
E_2 &= \left\{\frac{\nu_{21}^2}{E_1} + \frac{1}{8\overline{G}}\left[\frac{2(1-\psi_a)(\bar{\chi}-1)+(\chi_a-1)(\bar{\chi}-1+2\psi_a)\frac{\overline{G}}{G_a}}{2-\psi_a+\bar{\chi}\psi_a+(1-\psi_a)(\chi_a-1)\frac{\overline{G}}{G_a}}\right.\right.\\
&\quad \left.\left. +2\frac{\bar{\chi}(1-\psi_a)+(1+\psi_a\bar{\chi})\frac{\overline{G}}{G_a}}{\bar{\chi}+\psi_a+(1-\psi_a)\frac{\overline{G}}{G_a}}\right]\right\}^{-1},\\
G_{12} &= \overline{G}\frac{1+\psi_a+(1-\psi_a)\frac{\overline{G}}{G_a}}{1-\psi_a+(1+\psi_a)\frac{\overline{G}}{G_a}}, \quad G_{23} = \overline{G}\frac{\bar{\chi}+\psi_a+(1-\psi_a)\frac{\overline{G}}{G_a}}{(1-\psi_a)\bar{\chi}+(1+\bar{\chi}\psi_a)\frac{\overline{G}}{G_a}}, \\
\frac{\nu_{23}}{E_{22}} &= -\frac{\nu_{21}^2}{E_{11}} + \frac{1}{8\overline{G}}\left[2\frac{(1-\psi_a)\bar{x}+(1+\psi_a\bar{x})\frac{\overline{G}}{G_a}}{\bar{x}+\psi_a+(1-\psi_a)\frac{\overline{G}}{G_a}}\right.\\
&\quad \left. -\frac{2(1-\psi_a)(\bar{x}-1)+(x_a-1)(\bar{x}-1+2\psi_a)\frac{\overline{G}}{G_a}}{2-\psi_a+\bar{x}\psi_a+(1-\psi_a)(x_a-1)\frac{\overline{G}}{G_a}}\right],\\
\nu_{21} &= \bar{\nu} - \frac{(\bar{\chi}+1)(\bar{\nu}-\nu_a)\psi_a}{2-\psi_a+\bar{\chi}\psi_a+(1-\psi_a)(\chi_a-1)\frac{\overline{G}}{G_a}},
\end{aligned} \tag{5}$$

in which

$$\begin{aligned}
\bar{\chi} &= 3-4\bar{\nu},\\
\overline{\chi_a} &= 3-4\overline{\nu_a}.
\end{aligned} \tag{6}$$

### *2.2. Determine the effective thermal expansion coefficient of composite*

Similar to the elastic modulus, the thermal expansion coefficient of the three-phase composite materials were also identified in two steps: First, to determine the coefficient of thermal expansion of the effective matrix. The current paper uses the Duc's result from [18] for calculating the thermal expansion coefficient of effective matrix

$$\alpha^* = \alpha_m + (\alpha_c - \alpha_m)\frac{K_c(3K_m+4G_m)\psi_c}{K_m(3K_c+4G_m)+4(K_c-K_m)G_m\omega_c}, \tag{7}$$

in which $\alpha^*$ is the effective thermal expansion coefficient of effective matrix; $\alpha_m$, $\alpha_c$ are the thermal expansion coefficients of original matrix and particle, respectively.

Then, determining two coefficients of thermal expansion of the three-phase composite using formulas from [17] of Vanin

$$\begin{aligned}
\alpha_1 &= \alpha^* - (\alpha^*-\alpha_a)\psi_a E_1^{-1}\left[E_a + \frac{8G_a(\nu_a-\nu)(1-\psi_a)(1+\nu_a)}{2-\psi_a+\bar{x}\psi_a+(1-\psi_a)(x_a-1)\frac{\overline{G}}{G_a}}\right],\\
\alpha_2 &= \alpha^* + (\alpha^*-\alpha_1)\nu_{21} - (\alpha-\alpha_a)(1+\nu_a)\frac{\nu-\nu_{21}}{\nu-\nu_a}.
\end{aligned} \tag{8}$$

To numerical calculating, we chosen three phase composite polymer made of polyester AKAVINA (made in Vietnam), fibers (made in Korea) and titanium oxide (made in Australia) with the properties as in Table 1 [16].

The results of elastic modules of composite materials for different volume ratios of component materials written in (5) are given in Table 2 [7], in which 14 variant cases of different volume ratios of component three-phase composite materials respectively are given in Table 3 [7].

Fig. 1 illustrates the SEM images of the structure of the two-phase composite polymer with the based phase composed of the glassy polyester fiber (without the particles) in 1D (all the composite fibers are reinforced in one direction). Fig. 2 illustrates the structure of three-phase 1Dm in the presence of the $TiO_2$ particles.

From the above illustrations, it is obviously that the more the particles are doped, the finer the material is, in other words, the less the holes are. These results also mean that we can increase the elastic modulus as well as strengthens the penetration resistance of the materials by doping the particles. Figs. 1 and 2 show the SEM pictures of our proposed fabricated composite structures. These pictures are taken by ourself using the SEM instrumentation at the Laboratory for Micro-Nano Technology, University of Engineering and Technology, Vietnam National University, Hanoi. Also, we made these composite material samples in the Institute of Ship building, Nha Trang University.

## 3. Governing equations

Consider a three phase composite plate with midplane-symmetric. The plate is referred to a Cartesian coordinate system $x$, $y$, $z$, where $xy$ is the mid-plane of the plate and $z$ is the thickness coordinator, $-h/2 \leqslant z \leqslant h/2$. The length, width, and total thickness of the plate are $a$, $b$ and $h$, respectively.

In this study, the classical theory is used to establish governing equations and determine the nonlinear response of composite plates [13–15].