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Use of axiomatic/asymptotic approach to evaluate various refined theories for sandwich shells

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ABSTRACT

This paper evaluates refined theories for sandwich shells. Layer-wise and equivalent single-layer models (including zig-zag theories) are used with linear and higher order expansion in the thickness layer/shell direction for the displacement variables. So called asymptotic/axiomatic approach is employed to establish the effectiveness of each displacements terms for a given problems: that is the initial axiomatic expansion with all the terms related to the assigned order *N* is asymptotically reduced to a 'best' displacement models which has the same accuracy of the full model but with less terms. The various sandwich theories are conveniently formulated by using the unified formulation by Carrera (CUF) that leads to governing equations which forms are formally the same for the different sandwich shell theories. Accuracy of a given theory is established by fixing the sandwich shell in term of geometry, boundary conditions, layout of the face/core layers (including very soft-core cases) as well as by choosing a criteria to measure the errors. Two error criteria have been adopted which are related to a fixed point and to the maximum values of displacement/stress variables in the thickness shell direction. A number of problems have been treated and the related 'best' displacement model have been obtained. The effectiveness of the asymptotic/axiamotic problems is proved by comparing with available reference solutions. It has been found that the resulting reduced 'best' theories are very much subordinated to the considered problems. These changes by changing geometrically parameters as well as by adopting a different error criteria.

1. Introduction

Many problems are of interest for a better understanding of the mechanical behavior of sandwich shells. The books by [1–4] discuss the main aspects considered in the design, analysis, and construction of sandwich structures. Among these issues, our interest is in this work directed to compare shell theories able to provide accurate evaluations of stress/strain fields in each layer of the sandwich structures. On this respect, sandwich consists of one of the pioneering example of structure which demands amendments to classical theories originally developed for metallic one-layered structures. Two are the main reasons of that: sandwich structures are *multilayered* made (at least three layers); one of these layers (the core) has very high transverse shear deformability. Three-layers configuration demands for accounting so-called *zig-zag* (ZZ) for the distribution of displacement fields in the thickness directions as well as fulfillment of *interlaminar continuity* (IC) conditions for transverse shear and normal stresses at the two interfaces. Most of the contributions made in the last century on refined theories for multilayered beam/plate/shell structures have been, in fact, originated by early work on sandwich structure analysis, see [5].

Accurate analysis of stress/strain fields could be acquired by three-dimensional (3D) elasticity solutions, see [6,7]. However the use of two-dimensional 2D plate/shell models is preferred in most of the applications related to sandwich structures. 2D models are, in fact, more convenient than 3D ones in terms of required computational efforts. Various 2D models have been developed in the literature. These have been discussed in many texbooks [8], Ambartsumian [9–11], Librescu [12], and Reddy [13] and more recentely by Carrera et al. [14]. Over 800 references on sandwich structures are considered in Noor et al. [15] and in Reddy and Robbins [16], Carrera [5,17,18]. For our purpose 2D models sandwich structures could be classified by accounting for manner in which the description of the kinematic in the layers is made. Layer-wise models (LWMs) are those in which the number of the variables is independent in each layer. ZZ effect is intrinsically considered in the LWMs. The number of the unknown variables is kept independent by the number of the layers in case of Equivalent single layer models (ESLMs).

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In the last years Carrera and co-authors [21–26] have proposed a unified formulation (UF or CUF) for multilayered structures analysis. Classical models formulated were obtained as particular cases. LW and ESL models related to linear to fourth order expansion in the plate/shell thickness direction (z) were implemented. ZZ, IC, transverse shear, and normal strains effects were addressed. Navier-type closed form solutions as well as finite element solution were obtained [5]. Extensive application to sandwich structures plates and shells have been provided in Carrera et al. [18,27,19,20]. The application of UF permits an extensive evaluation of a large number of theories in both LW and ESLM framework.

In order to try to reduce the computational costs without losing accuracy a so called *axiomatic/asymptotic* approach has been recently established by Carrera and Petrolo [28]. CUF formulated theories, as definition, should be classified as axiomatic theories, that is the order of the expansion for the displacement variables is assumed 'a priori'. It is well known that in contrast to axiomatic approach, the asymptotic approach could be used. The latter expands the governing equations in terms of a perturbation parameter p of the structures (e.g. the length-to-thickness ratio) by leading to class-of-problems related to a set of governing equations which contain the whole contribution with the same order of magnitude with respect to p. Reviews and analysis on this approach with applications to plates and shells can be found in [29–32,15,16,33].

It has been previously shown that the introduction of high order terms in a given axiomatic model offers a benefit in terms of improved structural response analysis, with higher computational costs. The possibility to obtain accurate high order theories with less computational cost could be offered by evaluating the importance/effectiveness of each term of the expansion in the solution process. With that information a decision could be taken and the corresponding term could be retained (if relevant) or discarded (if not significant). By doing that the above axiomatic/asymptotic approach in [28] is obtained: the effectiveness of each displacement variable of a model is compared to a reference solution and the terms which do not influence the response are discarded. This technique was proposed in the finite element framework in [34,22,23]. The genetic-like algorithms were used in [35] to evaluate the importance of each displacement variables for FE plate models. The results of these works were presented in form of a diagram. That diagram was stated as 'Best Plate/Shell Theories curves'; it gives the minimum number of displacement variables versus the accuracy on a given stress or displacement parameter. Recent application to sandwich plates [36,37] have clearly shown that the choice of the most appropriate theories is very much dependent on the given problems, geometrical parameter of the sandwich structures, mechanical properties of faces and core, boundary conditions (mechanical and geometrical ones), to the unknowns variables (displacement, stress, strain components) used to measure accuracy as well as the criteria used to build the errors (one-points, multi-points, etc.). This work extend the previous finding to sandwich shell geometries.

2. Preliminary

In the following multilayered shells are considered. The shells have N_l layers and each layer is identified by an index k, which varies from 1 to N_l starting from the bottom. The reference system for a such structure is reported in Fig. 1 (the particular case of cylindrical shell has been drawn). α_k and β_k are the curvilinear orthogonal co-ordinate which coincides with the principal curvature lines. z_k denotes the rectilinear co-ordinate in the normal direction to the reference surface Ω_k. In the following a further dimensionless coordinate is introduced for each layer: $\zeta_k = \frac{2z_k}{h_k}$

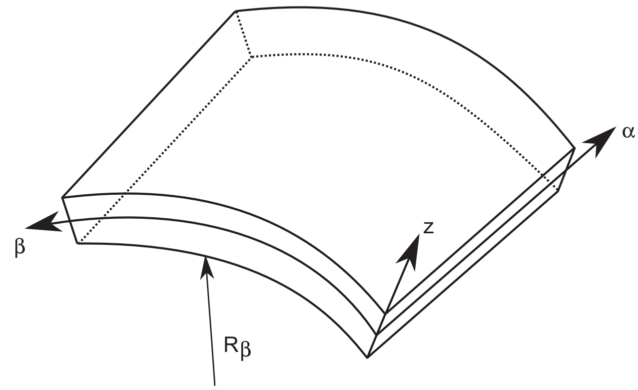

Fig. 1. Shell reference system.

where h_k is the thickness of the k-layer. For a given orthogonal system of curvilinear coordinates the infinitesimal length of a line element, the infinitesimal area of a rectangle lying on the surface Ω_k and the infinitesimal volume can be expressed respectively:

$$ds_k^2 = H_\alpha^k d\alpha_k^2 + H_\beta^k\, d\beta_k^2 + H_z^k\, dz_k^2, \quad d\Omega_k = H_\alpha^k H_\beta^k\, d\alpha_k\, d\beta_k,$$
$$dV = H_\alpha^k H_\beta^k H_z^k\, d\alpha_k\, d\beta_k\, dz_k \tag{1}$$

where

$$H_\alpha^k = A^k\left(1 + \frac{z_k}{R_\alpha^k}\right), \quad H_\beta^k = B^k\left(1 + \frac{z_k}{R_\beta^k}\right), \quad H_z^k = 1 \tag{2}$$

R_α^k and R_β^k are the curvature radii of the a k layer along the directions of α_k and β_k respectively. A_k and B_k are the coefficients of the first fundamental form of Ω_k.

3. Carrera unified formulation for shell theories

According to CUF the displacement field of a 2D structure can be described as:

$$\mathbf{u} = F_\tau \cdot \mathbf{u}_\tau \quad \tau = 1, 2, \ldots, N+1 \tag{3}$$

where $\mathbf{u}$ is the displacement vector and N is order of the expansion. As already described in the previous section the development of Eq. (3) can follow two different approaches: Equivalent Single Layer and Layer Wise. According to the former approach the behavior of a multilayered shell can be attributed to a single equivalent surface which sums up all the properties of a multilayered shell. According to the latter approach each layer of a multilayer shell presents instead its own displacement variables. It has to be highlight that as the number of layers increases the number of displacement variables required for Layer Wise approach increases as well but for a Single Layer Approach it remains the same.

In Equivalent Single Layer approach F_τ functions are polynomial functions of z defined as $F_\tau = z^{\tau-1}$. In the following the ESL models are synthetically indicated as EDN, where N is the expansion order. As example for an ED4 displacement field one has:

$$\begin{aligned} u_\alpha &= u_{\alpha_1} + z u_{\alpha_2} + z^2 u_{\alpha_3} + z^3 u_{\alpha_4} + z^4 u_{\alpha_5} \\ u_\beta &= u_{\beta_1} + z u_{\beta_2} + z^2 u_{\beta_3} + z^3 u_{\beta_4} + z^4 u_{\beta_5} \\ u_z &= u_{z_1} + z u_{z_2} + z^2 u_{z_3} + z^3 u_{z_4} + z^4 u_{z_5} \end{aligned} \tag{4}$$

Classical Lamination Theory, CLT and First order Shear Deformation Theory FSDT [12], can be considered as special cases of full linear expansion (ED1). An improvement of an ESL model follows from the introduction of Murakami's Zig-Zag function:

$$\mathbf{u}_{ZZ} = (-1)^k \zeta_k \cdot \mathbf{u}_Z \tag{5}$$