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# A particular implementation of the Modified Secant Homogenization Method for particle reinforced metal matrix composites

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ABSTRACT

The stress–strain relationship for Particle Reinforced Metal Matrix Composites (PMMCs) in the nonlinear regime has been frequently assessed by means of the Modified Secant Homogenization Method. This nonlinear Homogenization uses a Linear Elastic Homogenization scheme to calculate the Secant Compliance tensor of the composite in terms of the known Secant Compliance tensors of the composites' constituent phases.

The subject of the present work is the development of a particular implementation of the Modified Secant Homogenization Method for the case of PMMCs using, as the required underlying Linear Elastic Homogenization scheme, the Halpin–Tsai equation. The developed implementation, valid for PMMCs of geometrically isotropic microstructure, results in a relatively simple iterative procedure for the estimation of the nonlinear macroscopic stress–strain response. It has only two explicit parameters: the reinforcement volume fraction $F$ and the '$s$' parameter of the Halpin–Tsai equation, which carries implicitly information about particle aspect ratio and orientation.

The proposed scheme is applied to the prediction of the uniaxial hardening curve and to the study of the influence of macroscopic hydrostatic stress on composite's yield.



## 1. Introduction

Particulate Reinforced Metal Matrix Composites (PMMCs) have been the subject of significant research interest during the last sixty years. These materials are usually constituted by an elasto-plastic metal matrix as a continuous phase in which elastic particles are embedded. Particles are considered a discontinuous "reinforcement", as they are normally chosen to be stiffer than the matrix. For matrix, alloys of aluminum, magnesium or titanium can be used, while particles are generally made of ceramic materials such as silicon carbide or alumina.

The mechanical behavior of PMMCs depends of the constitutive properties of the constituent phases and also of microstructural features such as particle volume fraction, particle shape, particle orientation and spatial distribution of the particles within the matrix. A large number of methods and models to study the mechanical behavior and properties of PMMCs have been proposed. They can be grouped into four broad families: methods based on mean-field theory, methods based on homogenization by asymptotic expansion, variational methods, and methods based on cell-analysis. As the literature in the field is very extensive, a full bibliographic overview is out of the scope of the present work. Consequently, only a small set of references are mentioned here, involving review books and some of the relevant original papers. General review books about PMMCs are [1–4]. Important books from the perspective of micromechanical analysis are [5–8].

Although the context of the present work is in the mean field approach, tangentially some aspects of cell modeling and variational bounds are touched. In mean field approaches, it is assumed that volume averages of the local stress and strain tensor fields inside a Representative Volume Element (RVE) of material are valid representative measures of the macroscopic stress and strain tensors. In this way, effective mechanical properties of the composite are defined in the form of a fourth order compliance (or stiffness) tensor which relates the average stress with the average strain. In linear elasticity, these constitutive tensors of effective elastic properties are independent of strain. In the nonlinear regime, however, the effective constitutive tensors change with macroscopic stress or strain. As a result, the characterization of the effective mechanical behavior has to be made in terms of a secant or tangent compliance (or stiffness) tensor.

Because it has been shown that tangent approaches tend to predict a stiffer response of the composite (see for instance [5,9,10]), the stress–strain relationship for PMMCs in the nonlinear regime has been frequently assessed in the literature by means of a Secant Homogenization Method, usually in its Modified form by Suquet [11], who also showed the connection of the method with the nonlinear variational estimates of Ponte Castañeda [12]. Application

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examples of the Modified Secant Method can be found in [13–15]. This Homogenization Method for the nonlinear regime uses a Linear Elastic Homogenization scheme to calculate the Secant Compliance tensor of the composite in terms of the known Secant Compliance of the constituent phases.

It is clear, thus, that an implementation of the Modified Secant Homogenization scheme requires the availability of a suitable Linear Elastic Homogenization scheme. A large number of such schemes have been proposed in the literature for composites reinforced with spherical or ellipsoidal particles which are, generally, based on estimations of the stress localization tensor made using the well known Eshelby tensor [16], as in the Mori–Tanaka model [17,18], among others. These Linear Elastic methods cannot rigorously be used, however, for non-ellipsoidal particles, or particles with complex shape, for which no Eshelby tensor is known or easily estimated.

Zahr Viñuela and Pérez-Castellanos [19] proposed a Linear Elastic Homogenization method for PMMCs where the reinforcing phase consists of particles of varying aspect ratios embedded in an elastic–plastic matrix. This approach is based on computational micromechanics in the form of multi-particle cell modeling, and can be summarized as follows: Multi-particle cells representing geometrically isotropic microstructural scenarios were built and analyzed by Finite Element for a range of values of particle volume fraction $F$; particles with edges and irregular shape were modeled as prismatic particles in both, random and controlled orientation. The numerical results provided values of homogenized elastic constants in excellent agreement with its experimentally measured counterparts, for a composite material with an aluminum alloy matrix reinforced with SiC irregular particles. Subsequently, the authors used the well known Halpin–Tsai expression [20] – in a modified form proposed by Halpin and Kardos [21] – as a fitting expression for computationally obtained homogenized elastic constants. That work provided for a set of values of the Halpin–Tsai '$s$' parameter for different microstructural scenarios: some of these are improved '$s$' values applicable to microstructural scenarios already analyzed by Halpin and Kardos, while other represent a completely new set of values applicable to microstructural scenarios non-covered previously in the literature, most notably, the geometrically isotropic cases of randomly oriented prismatic particles with large, small and unit particle aspect ratio. The main reason for Zahr and Pérez-Castellanos to use prismatic particles instead of ellipsoidal ones in their multi-particle cell analysis is that many real particulate composites contain particles of irregular shape with "hard" edges, so prismatic particles were considered to be a better approximation than spherical or ellipsoidal. The Halpin–Tsai equation proved a useful and easy to calibrate fitting function for the problem of homogenization of elastic constants in linear elasticity.

The subject of the present work is the development of a particular implementation of the Modified Secant Homogenization Method for the case of PMMCs using, as the required underlying Linear Elastic Homogenization scheme, the scheme proposed by Zahr and Pérez-Castellanos. The proposed implementation results in an iterative procedure for the estimation of the nonlinear macroscopic stress–strain response in which there are only two explicit parameters: the reinforcement volume fraction and the '$s$' parameter of the Halpin–Tsai equation which, in turn, carries implicitly the information of particle aspect ratio and orientation.

The method is then applied to the prediction of the uniaxial hardening curve of a PMMC characterized by a geometrically isotropic microstructure where the reinforcing particles are randomly oriented inside the matrix, and for which, several stress–strain curves are obtained for different volume fractions. The influence of the macroscopic hydrostatic and Mises stresses on the yield condition of the composite is also studied.

## 2. The modified secant model

In the present section, a summary of the Secant Homogenization Method by Suquet is presented in a notation suitable to the particular kind of materials considered in this work, namely PMMC's made up of particles embedded in a metal matrix. For the subsequent development, the following three assumptions will be used and its consequences described:

i. Reinforcing particles follow a constitutive behavior which is linear, elastic and isotropic. This discontinuous reinforcement represents the "hard" phase of the composite.
ii. The matrix phase, which represents the continuous and "soft" phase, has a nonlinear constitutive behavior, with plastic strain and also J2 isotropic hardening. This constitutive behavior will be represented using Deformation Theory of Plasticity (a secant theory using total strains rather than incremental strains). Due to this, the analysis is restricted to proportional loading.
iii. The spatial distribution of particles within the matrix is geometrically isotropic.

According to [19], as the constitutive behaviors of the constituent phases is isotropic, and as the spatial distribution of the reinforcing particles within the matrix shows geometric isotropy, the constitutive behavior of the composite can also be considered isotropic, showing – as the matrix – plastic strain and isotropic strain hardening. It must be noted, however, that only the "isotropic" feature of the hardening of the matrix phase is transferred to the composite. The "J2" aspect of the isotropy of the matrix hardening does not transfer to the composite, as will be shown in subsequent sections.

In what follows, inertial and body forces are neglected. In this situation, let $V$ be a Representative Volume Element (RVE) of the composite material, which results from the union $V = \mathrm{U}(V_r)$, where $\boldsymbol{r} = \boldsymbol{m}$atrix or $\boldsymbol{p}$articles, $V_m$ and $V_p$ being non-intersecting regions of $V$ occupied, respectively, by matrix and particle materials. The full problem to be solved is formed by the set of Eqs. (1)–(5), where the variables are as follows: $\sigma$, $\varepsilon$ and $u$ are the stress, strain and displacement fields, respectively, inside the RVE, while $\Sigma$ and $\underline{\varepsilon}$ are the representative measures of the macroscopic or composite's stress and strain tensors.

It is worth noting that Eqs. (1)–(3) represent a *local problem*, while Eq. (4) represents a *homogenization problem* associated to the *local problem*, with Eq. (5) being a link between both, the *local* and *global* problems, as the brackets in this equation represents volume averages over $V$.

$$\textit{Local internal equilibrium}: \quad \nabla\sigma(x) = 0 \quad x \in V \tag{1}$$

*Strain—displacement compatibility* :

$$\varepsilon(x) = \frac{1}{2}\{\nabla \otimes u + (\nabla \otimes u)^T\} \quad x \in V \tag{2}$$

*Local constitutive equation* :

$$\varepsilon(x) = G_r(\sigma(x)) \quad x \in V_r \quad \text{with } \boldsymbol{r} = \boldsymbol{m}atrix,\ \boldsymbol{p}articles \tag{3}$$

$$\textit{Homogenization}: \quad \underline{\varepsilon} = G(\Sigma) \tag{4}$$

*Link between local and macroscopic variables* :

$$\Sigma = \langle\sigma\rangle_V; \quad \underline{\varepsilon} = \langle\varepsilon\rangle_V \tag{5}$$

In Eq. (3), $G_r$ is a function which represents in general the constitutive behavior of each constituent phase. The particular form of the function $G_r$ which relates the stress and strain tensors in each point inside each phase is considered a data for the problem. Also, it must