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Linear instability analysis of planar non-Newtonian liquid sheets in two gas streams of unequal velocities

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ABSTRACT

Linear stability theory is applied to study the breakup process of a non-Newtonian liquid sheet subjected to non-zero unequal gas flow on both sides of the liquid sheet. The unequal non-zero velocities of the gas streams on both sides are considered in this temporal instability analysis. The dispersion relation between the growth rate of disturbances and the wave number of disturbances is derived. Then a parametric study of the instability of the liquid sheet is made. The emphasis of the paper is to study the effects of various velocities of the upper and lower gas streams. It is found that the larger velocity difference across each interface enhances the instability of the sheets. But the enhanced extent could be different. The instability is primarily determined by the larger one between the velocity differences across the two liquid–gas interfaces for para-sinuous mode and the smaller one for the para-varicose mode. The influences of the density ratio, surface tension and liquid viscosity on the instability of the planar sheet are also included in this paper.

1. Introduction

The breakup of liquid sheets in a gaseous ambient is of considerable scientific importance and has also been used in several industrial applications including chemical combustion, gas turbines, diesel engines, spray cooling, surface coating, and medicine. Savart [1] studied the dynamics of a liquid sheet first in 1833. Dorman [2] and Fraser et al. [3] were the first to describe the breakup and drop formation of planar sheets. Over the past century, the breakup mechanism of the liquid sheets has been investigated extensively.

The linear stability theory can evaluate the beginnings of instability of the liquid sheet; it offers a good prediction for the onset of instability of the liquid stream. Moreover, the breakup length and droplet radius can be estimated using linear stability analysis. Therefore, extensive linear studies have been conducted on the instability of the liquid sheets.

The linear instability of a planar liquid sheet with constant thickness was analyzed by Squire [4] and Hagerty and Shea [5]. They considered the instability with respect to the classical temporal growing single Fourier component of the disturbance. In general, there are two independent modes of unstable waves which exist on the liquid–gas interface: an antisymmetric wave or the so-called sinuous mode and a symmetric wave or the varicose mode. While the distortion on two liquid–gas interfaces is in-phase, it is referred to as the 'sinuous disturbance wave'. While the distortion on two interfaces is out-of-phase, it is referred to as the 'varicose disturbance wave'. Most studies indicate that sinuous waves are more unstable than varicose waves. However, Li and Tankin [6] and Rangel and Sirignano [7] presented a more complex picture. From Li and Tankin [6], it is clear that, when the Weber numbers (the ratio of inertial and interfacial surface tension forces) is small enough, sinuous waves even become neutrally stable while varicose waves remain unstable. Thus, the conclusion that sinuous waves are more unstable than varicose waves is reasonable only for large Weber numbers. The spatial instability of planar sheets was considered in connection with a process called 'curtain coating'. Brown [8] observed the 'curtain problem' experimentally. Lin [9] made a linear temporal and spatial stability analysis of a viscous liquid curtain in a void. Ibrahim and Akpan [10] presented a fully three-dimensional linear analysis of a plane viscous liquid sheet in an inviscid gas medium. Lin et al. [11] extended the study to include the effects of an ambient gas. Sirignano and Mehring made a review of the previous fundamental mechanisms of the distortion and disintegration of liquid streams [12].

In practice, the fluid in some applications such as polymer solutions, gels, food dispersions, paints, inks, and other complex fluid formulations may exhibit non-Newtonian rheological behavior. The mechanisms of non-Newtonian liquid sheets are of both practical and theoretical interest. However, very little is known about the instability and breakup of non-Newtonian liquid sheets [13–16]. A linear stability analysis [14,15] shows that non-Newtonian liquid sheets have a higher growth rate than Newtonian liquid sheets for both varicose and sinuous disturbances, indicating that non-Newtonian liquid sheets are more unstable than Newtonian liquid sheets. It

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has been discovered that the maximum growth rate of sinuous disturbances is always larger than that of varicose disturbances, while the dominant wave number of sinuous disturbances is always smaller than that of varicose disturbances. This indicates that sinuous disturbances always prevail over varicose disturbances for non-Newtonian liquid sheets. Parthasarathy [17] conducted a linear spatial instability analysis of slurry sheets subjected to gas flow. The slurry behavior were described with viscoelastic model. They stated that for viscoelastic fluids, the Deborah number De = $\theta h/U$ played an important role in the wave growth and the gas velocity acted as a destabilizing agent, where θ is the magnitude of relaxation time of viscoelastic fluid, h is half thickness of the sheet and U is the liquid sheet velocity.

In the theoretical works cited above, the ambient gas on the two sides of the sheet is considered to be stationary. The two independent modes of disturbance found are strictly sinuous and varicose. However, it is well known that the gas velocities on the two sides of the sheet can be different in many practice applications referred to as twin-fluid (e.g. air-assist and air-blast) atomization, which is extensively used in many industrial processes including fuel preparation in jet engines and spray drying processes. Li [18] performed a temporal stability analysis of liquid sheet in gas streams of unequal velocities where one gas stream was stationary and it was found that there were another two modes of instability, which are para-sinuous (when phase difference between two interfaces is close to zero) and para-varicose (when phase difference between two interfaces is close to π). But in fact such liquid sheet is subjected to non-zero velocities on both sides of gas stream. Nath et al. [19] conducted a temporal stability analysis of a planar liquid sheet sandwiched between two gas streams of unequal velocities. In their study, the liquid is inviscid. Witherspoon & Parthasarathy [20] performed a spatial instability analysis of a viscous liquid sheet that was subjected to equal and unequal gas velocities and documented the effects of gas velocity on the growth of disturbances on the liquid sheet for a range of Reynolds number, Weber numbers and density ratios.

To investigate the breakup characteristics of planar non-Newtonian liquid sheets, the co-rotational Oldroyd eight-constant model is used to describe the viscoelastic characteristics of the non-Newtonian fluids; and then a linear stability model is built up based on the linear stability analysis method. The emphasis of this paper is to analyze the influences of the different and non-equal velocities of two gas streams on both sides of non-Newtonian sheets. The effects of various flow parameters, such as liquid viscosity, gas to liquid density ratio and surface tension, on instability are also included. Finally, some helpful conclusions are drawn.

2. Linear analysis model

A two-dimensional sheet of non-Newtonian liquid moving through inviscid gas is considered. The liquid flow is uniform with a velocity U and a thickness $2a$. The coordinates are chosen so that the x-axis is parallel to the direction of the liquid sheet flow, and the y-axis is normal to the liquid sheet, with its origin located at the middle plane of the liquid sheet. Fig. 1 shows a schematic diagram of a moving liquid sheet under sinuous and varicose disturbances. If the disturbances keep on growing, the sheet will breakup into filaments.

The governing equations of the liquid motion in a sheet are the conservation laws of mass and momentum, as given below:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho v) = 0 \tag{1}$$

$$\rho \frac{\mathrm{D}\boldsymbol{v}}{\mathrm{D}t} = \rho \left(\frac{\partial}{\partial t} + \boldsymbol{v} \cdot \nabla \right) \boldsymbol{v} = -\nabla p - \nabla \cdot \boldsymbol{\tau} \tag{2}$$

where t is time, ρ is the density of the liquid, $\boldsymbol{v}$ is the liquid velocity vector, and $\boldsymbol{\tau}$ is the viscous shear stress tension of the liquid. For a two-dimensional liquid sheet, the velocity vector $\boldsymbol{v}$ has only two components, $\boldsymbol{v} = (u, v)$.

It is necessary to adopt a rheological equation of state. In the present study, the co-rotational Oldroyd eight-constant model is used to describe the viscoelastic liquid state, which has the following general constitutive equation in the objective reference frames [21].

$$\begin{aligned} &\boldsymbol{\tau} + \lambda_1 \frac{\mathrm{D}\boldsymbol{\tau}}{\mathrm{D}t} + \frac{1}{2}\mu_0 (tr\boldsymbol{\tau})\dot{\gamma} - \frac{1}{2}\mu_1 (\boldsymbol{\tau} \cdot \dot{\gamma} + \dot{\gamma} \cdot \boldsymbol{\tau}) + \frac{1}{2}\nu_1 (\boldsymbol{\tau} : \dot{\gamma})\boldsymbol{\delta} \\ &\quad = -\eta_0 \left[\dot{\gamma} + \lambda_2 \frac{\mathrm{D}\dot{\gamma}}{\mathrm{D}t} - \mu_2 (\dot{\gamma} \cdot \dot{\gamma}) + \frac{1}{2}\nu_2 (\dot{\gamma} : \dot{\gamma})\boldsymbol{\delta} \right] \end{aligned} \tag{3}$$

where

$$\dot{\gamma} = \nabla \boldsymbol{v} + (\nabla \boldsymbol{v})^T \tag{4}$$

$$\boldsymbol{\omega} = \nabla \boldsymbol{v} - (\nabla \boldsymbol{v})^T \tag{5}$$

$$\frac{\mathrm{D}\boldsymbol{\tau}}{\mathrm{D}t} = \frac{\partial \boldsymbol{\tau}}{\partial t} + (\vec{v} \cdot \nabla)\boldsymbol{\tau} + \frac{1}{2}(\boldsymbol{\omega} \cdot \boldsymbol{\tau} - \boldsymbol{\tau} \cdot \omega) \tag{6}$$

$$\frac{\mathrm{D}\dot{\gamma}}{\mathrm{D}t} = \frac{\partial \dot{\gamma}}{\partial t} + (v \cdot \nabla)\dot{\gamma} + \frac{1}{2}(\boldsymbol{\omega} \cdot \dot{\gamma} - \dot{\gamma} \cdot \boldsymbol{\omega}) \tag{7}$$

Here $\dot{\gamma}$ is the strain tensor, $\boldsymbol{\omega}$ is the vorticity tensor, D/Dt is the co-rotational derivative, η_0 is the zero shear viscosity, λ_1 is the stress relaxation time, λ_2 is the deformation retardation time of the liquid, and $\boldsymbol{\delta}$ is the unit tensor. The quantities μ_0, μ_1, μ_2, ν_1, and ν_2 are time constants.

In this paper, gas around the liquid sheet is assumed to be inviscid, that is, $\boldsymbol{\tau}_g = 0$. The governing equations for the gas phase are expressed similarly to those for the liquid phase, as follows:

$$\frac{\partial \rho_{g,i}}{\partial t} + \nabla \cdot (\rho_{g,i} \boldsymbol{v}_{g,i}) = 0 \tag{8}$$

$$\rho_{g,i} \frac{\mathrm{D}\boldsymbol{v}_{g,i}}{\mathrm{D}t} = \rho_{g,i} \left(\frac{\partial}{\partial t} + \boldsymbol{v}_{g,i} \cdot \nabla \right) \boldsymbol{v}_{g,i} = -\nabla p_{g,i} \tag{9}$$

where ρ_g is the density of the gas, $\boldsymbol{v}_g = (u_g, v_g)$ is the gas velocity vector and the subscript i = 1 and 2 correspond to the upper and lower gas stream, respectively.

The solutions of the above governing equations must satisfy the boundary conditions at the liquid interface $y = Y_i(x,t)$, where i = 1 for the upper interface and i = 2 for the lower interface. One condition is the kinematic condition that a particle of fluid on the surface moves with the surface so as to remain on the surface, in other words the velocity component normal to the interface is continuous across the interface, i.e.

$$v = \frac{\partial Y_i}{\partial t} + u \frac{\partial Y_i}{\partial x} \quad \text{at } y = Y_i(x,t) \tag{10}$$

$$v_{g,i} = \frac{\partial Y_i}{\partial t} + u_{g,i} \frac{\partial Y_i}{\partial x} \quad \text{at } y = Y_i(x,t) \tag{11}$$

The second condition considers the balance between the surface stresses on both sides of the interface, including the pressure jump across the interface due to surface tension. These dynamic boundary conditions require that

$$\tau_{xy} + (\tau_{xx} - \tau_{yy})(\partial Y_i/\partial x) - \tau_{xy}(\partial Y_i/\partial x)^2 = 0 \quad \text{at } y = Y_i(x,t) \tag{12}$$

$$\begin{aligned} &p + \tau_{yy} - \frac{2\tau_{xy}(\partial Y_i/\partial x)}{1 + (\partial Y_i/\partial x)^2} + \frac{(\tau_{xx} - \tau_{yy})(\partial Y_i/\partial x)^2}{1 + (\partial Y_i/\partial x)^2} - p_{g,i} + \sigma \nabla \cdot \vec{n} = 0 \\ &\text{at } y = Y_i(x,t) \end{aligned} \tag{13}$$

where $\boldsymbol{\sigma}$ is the surface tension, $\vec{n}$ is the local unit normal vector of the interface pointing into the gas phase and $\nabla \cdot \vec{n}$ is the local surface curvature.